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Investigation of cut control equations in the gas centrifuge cascades

A. Norouzi[1], M. Khajenoori[*1], J. Safdari[2], S. Yousefi Nasab[1], M. Mallah[2], J. Karimi Sabet[2]

AFFILIATION

[1]Iran Advanced Technologies Company, Atomic Energy Organization of Iran, Tehran, Iran
[2] Materials and Nuclear Fuel Research School, Nuclear Science and Technology Research Institute, Tehran, Iran

[*]**Author to whom correspondence should be addressed**: mkhajenouri@aeio.org.ir

ABSTRACT

Stage cut control and simulation are the most important aspects in the optimum binary mixture or multi-component multi-objective cascades. Numerical investigation revealed that by controlling the cut of a separation cascade, defined as the ratio of the product rate to the feed rate, it is always possible to separate a multi-component mixture into two specified groups of components, a light group, and a heavy group, in just one separation run. In this paper, the equations related to the cut control are introduced and it is proposed that for controlling stage cuts, putting one valve in the product section of each stage is enough. By solving the set of non-linear equations related to the machine behavior, valve, and pressure drop in the pipelines and junctions, the valve setting for each stage can be obtained. In the end, some examples of an optimal cascade are studied and valve setting parameters are obtained.

Keywords: Gas centrifuge, Optimum Cascade, Pressure, Cut Control

I. INTRODUCTION

In the optimum binary mixture cascade or multi component model cascades, calculating of cuts is one of the essential parts of work. In the optimum cascade the total flow reaches to its minimum value and it can be better than ideal cascade [1-3]. The cut values of stages with no equipment for controlling the stage cut is depend on the external (pressure of the feed, product and waste lines) and internal parameters (machine characteristics) of the gas centrifuge. In this state, the cascade operation moves away from the ideal or optimal condition. The cascade with cut control equipment can be set in the optimal value or can switch between model cascades in the multicomponent case. By controlling the cut of a separation cascade, it is possible to separate a multicomponent mixture into two groups of components [4]. The studies on the optimal cascade idea continue and extend every day. In 1997 and 1998 Palkin [3, 5] presented dependence of separation factor to two variables cut and feed as a correlation and illustrated that the total number of centrifuges in the ideal cascade is higher than its value in the optimal cascade. In 2010 Norouzi et al. applied the outcome of the purely axial flow model [1] to the concentration equation in single gas centrifuge for a binary isotope mixture to achieve a realistic function for α in relation to θ and f and relax the restriction of Palkin's model. In 2012 Palkin and Igoshin proposed a method for calculating and optimizing a cascade of gas centrifuge with an arbitrary scheme in an independent quantity of cut which separation factor is depend on cut and feed flow rate [6]. In 2014 Borisevich et al. [2] proposed that it is possible to find the optimum parameters of a cascade that operates in the minimum total flow. In general behavior the separation work, δU, is a function of F, α and θ [7], however experimental data show that δU is a function of the feed flow F, θ and pressure in the product line, Pp [8,9]. In fact, the external parameters influence on the internal parameters (F, α and cut θ) in the cascade. The external parameters of a single centrifuge are defined by the pressures in the feed P_F, product P_P and waste P_W or by the feed (F), the product (P) and waste flow (W). In 2005 Andrade et al. [8] study an isotope separation test under different operational conditions, defined by three process variables (the feed flow F, the pressure of the product header P_P and the cut). They generate several groups of data and each of them is denominated a separation experiment. Mathematical analysis of these experiments can create a function or heuristic method to describe the single machine behavior. The single gas centrifuge function (SGCF) can be used to adjust cuts of stages.

Ezazi *et al.* proposed a cascade design method using artificial bee colony algorithm to find multicomponent mixture separation cascade optimal parameters. Therefore, the real coded artificial bee colony was developed and the waste and product concentrations of a target component and the total interstage flow of the cascade were selected as the objective function main items [10]. Imani *et al.* investigated the effect of holdup of pipes and stages as well as the shape of cascade in separating the multi-component isotopes in NFSW cascades. These cascades are optimized by the Particle Swarm Optimization (PSO) algorithm with an advanced objective function to evaluate the effect of the cascade shape. Their results showed that the optimal tapered shape is superior over the square shape in separating the middle component of Xenon [11].

The object of this work is to find a way to adjust the known cut values in the optimum or multicomponent cascades. For this reason, we presume a valve for each product section and present a set of non-linear equations related to it. By solving these equations, the valve setting for each stage is obtained. The hypothetical SGCF are used to calculate the behavior of the cascade. Solving the equations shows that, it is possible to control the stage cuts by one valve in the product section.

II. CASCADE STRUCTURE AND STAGE EQUATION

A simple separation cascade consisting of separating units connected in series is shown in Figure 1. The head stream of a stage goes to feed the next upper stage and its tail stream is recycled at the inlet of the next lower stage [1]. The cascade receives a feeding material, with a molar flow rate F (g/h) and composition X_F, and delivers a product stream, with molar flow rate P and composition X_P and waste stream with molar flow rate W and composition X_w.

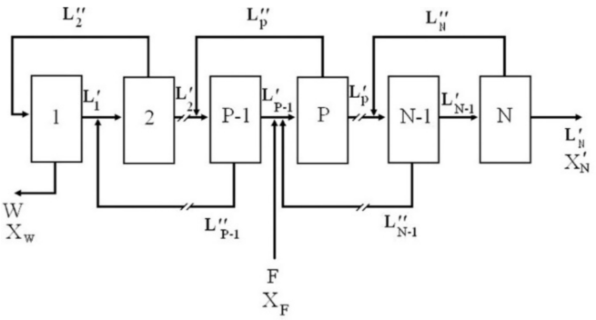

Fig. 1. Cascade of centrifuges with n stages.

These six parameters defining outer operating conditions are called outer parameters of a cascade. Stages are consecutively numbered from one to N with stage number p for external feed injection. Outer parameters must satisfy balances on material and on desired component over the entire cascade, that is:

$$L' = \theta_s L_s, \tag{1}$$

$$L'' = (1-\theta_s)L_s, \quad s = 1,...,N \tag{2}$$

Inner parameters completely defining the cascade are flow rates and compositions of feed, head and tail of each stage as well as the total number of stages. The inner parameters of the cascade are determined as functions of stage equation, taking into account outer parameters.

In a symmetric cascade with N stages and having that θ_s for every stage, the flow rate of pipes can be obtained by the following set of material balance equations:

$$L'_s = \theta_s L_s, \qquad s = 1,...,N \tag{3}$$

$$L'' = (1-\theta_s)L_s, \quad s = 1,...,N \tag{4}$$

$$L''_s = (1-\theta_s)L_s, \quad s = 1,...,N \tag{5}$$

$$L_N = L'_{N-1}, \tag{6}$$

$$L''_{s+1} = L'_s + L''_{s+1}, \quad s = 1,...,N-2 \quad s \neq p \tag{7}$$

$$L_1 = L''_2, \tag{8}$$

where L_S, L'_S and L''_S are the input flow rate, product and waste streams of an arbitrary stage s respectively. The number of above equations is equal to a number of our unknowns.

Obtaining feed and waste flow rate of stages, one can obtains concentration distribution along the cascade by:

$$\frac{X'_s}{1-X} = \alpha_i \frac{X''_s}{1-X''_s}, \qquad S = 1,2,....,N \tag{9}$$

$$X_s = \theta_s X'_s + (1-\theta_s)X''_s, \qquad S = 1,...,N \tag{10}$$

$$L_{s+1}X_{s+1} = L'_s X'_s + L''_{s+2}X''_{s+2}, \tag{11}$$

$$X'_{N-1} = X_N, \tag{12}$$

$$X_1 = X''_2, \tag{13}$$

where X_s, X'_s and X''_s are the composition of desired isotope in the feed, product and waste stream of stages respectively [8].

A. Functional Simulation of Gas Centrifuge

In general form the cut and separation factors may assume to be dependent of three streams pressures (product, waste and feed):

$$\theta = f(P_p, P_w, P_F), \tag{14}$$

$$\alpha = f(P_p, P_w, P_F), \tag{15}$$

$$\alpha = \exp(\alpha_0 + \alpha_1\theta - \alpha_2\theta^2) f^{-\alpha_s}, \tag{16}$$

where $\alpha_0 = 1.2,\ \alpha_1 = 1.8,\ \alpha_2 = 2.2,\ \alpha_3 = 0.4$ At high rates of rotation, the gas is compressed into a narrow annular region near the cylinder wall and a good vacuum is established in the inner region of the centrifuge. The feed gas is introduced from a hole in the pipe which is located in the axis of the gas centrifuge, therefore one side of the feed pipe approximately have a constant pressure and is separated from the product and waste scoops that are located in the dense region. So, the feed pipe is related to one side of pipe. The dependence of pressure of feed pipe on feed is exposed in eq. (17).

$$P_s = f(L_s). \tag{17}$$

B. Equations of Cut Control

According to Darcy–Weisbach equation, the pressure loss in the constant hydraulic diameter due to viscous effects ΔP_0 is proportional to the length of pipe and density of the fluid.

$$\frac{\Delta P_0}{L} = f_D \frac{\rho}{2} \frac{\overline{v}}{D}, \tag{18}$$

where, L and D are length and diameter of pipe, f_D is Darcy friction factor, and υ is the mean flow velocity. The joined stream lines in the symmetric cascade may have the difference between pressures

$$\Delta P_{P(s-1)} + P_{P(s-1)} = \Delta P_{w(s+1)} + P_{w(s+1)} = P_{F(s)} = P_s, \tag{19}$$

where ΔP is the pressure loss due to the difference of the length, instruments and junctions. The full set of equations of pressure losses have been introduced by previous work [12]. The valve equation may be written as below:

$$m = BC_d A P_u \times \sqrt{\frac{\gamma}{RT}\left(\frac{2}{\gamma-1}\right)\left(\left(\frac{P_d}{P_u}\right)^{2/\gamma} - \left(\frac{P_d}{P_u}\right)^{(\gamma+1)/\gamma}\right)}, \tag{20}$$

where P_u is the upstream pressure (torr), P_d is the downstream pressure (torr), T is temperature of flow (K), R is gas constant ($J{\cdot}K^{-1}{\cdot}mol^{-1}$), B is dimensional/units conversion constant, and A is the cross-section area (cm^2). In this work we consider the value A as an unknown variable. The upstream and downstream pressures of the valve can be related to the P_P and P_W by following equations:

$$P_{us} = P_{ps} + \Delta P_{s(p-U)}, \tag{21}$$

$$P_{ds} = P_{fs+1} + \Delta P_{s(F-d)}, \tag{22}$$

where i is the stage number and $\Delta P_{i(P-U)}$ is the pressure loses due to friction in the i^{th} stage between P_P and upstream pressure of the valve.

Using equations (3-8) the flow rate of pipes can be obtained and the pressure losses in stages ΔP_s can be calculated by [12] or Darcy equations. So, the unknown parameters are the A_s and pressures in the product P_{Ps}, waste P_{Ws} and feed P_{Fs} of each stage. In General, there are 4N unknown variables and 4N equations for solving them. The equations (14), (17), (19) and (20) should be used for solving the 4N variables. We can set the pressure in the waste of the cascade, P_W, as an input parameter and remove one valve equation; by this method the interior valve should be set. The algorithm for solving the core equations is shown in Fig. 2.

III. RESULT AND DISCUSSION

In order to show the capability of the presented equations, in this section we present the solution of two test problems. The first case is the output of an optimum cascade and the second is a 10-stage cascade by constant cut. In this work the value of cut considered by following function [5]:

$$R = \sqrt{0.1(P_p - 1)^2 + 0.1(P_w - 10)^2} + \varepsilon, \tag{23}$$

$$C_F = \alpha \times f_i + b, \tag{24}$$

$$\theta = f(P_p, P_w, P_F) = C_F \times \sin(R)./R + 0.3. \tag{25}$$

where a=-1/900 and b=0.61111 and fi is the feed flow rate for single gas centrifuge. The function is chosen in the form that increasing in the product pressure or decreasing in the waste pressure leads to reduce the cut and cut should have acceptable value. Fig. 3 shows the cut dependence on the pressures.

The first case is the optimum cascade which is presented by palkin and is optimized by Norouzi [1], In order to develop the concept of cut control without the difficulty associated with detailed hydrodynamic calculations, we suppose that the pressure loses due to friction in the pipe is negligible. The relationship between feed flow rate and feed pressure can be written as:

$$P_F = 0.12 \times f_i + 1.3. \qquad (26)$$

where the amount of discharge coefficient $Cd = 0.1, B = 3.6 \times 10^6$, temperature T=300 *K*, and P_W=3.2. Table 1 presents the parameters and valve cross sections (A) of the optimum cascade.

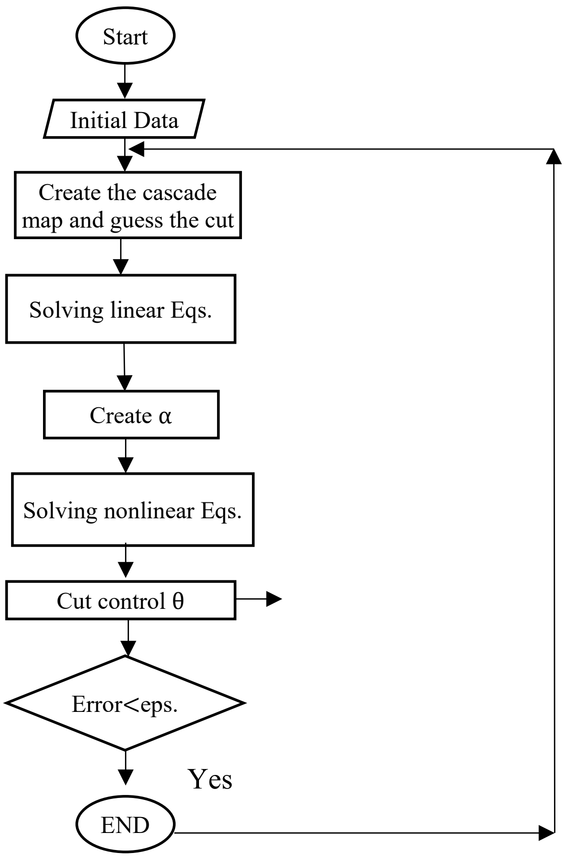

Fig. 2. Algorithm of calculation core

The second problem is a cascade with 10 stages which the feed is injected from 7th stage, the input parameters are listed in Table 2. Tables 3 and 4 present the parameters and valve cross sections, A, for different cut values. As can be seen, by changing the cut, the amount of feed and product pressure changes in the same cascade with the same number of machines and the same valve cross sections. Also, in the higher cut, the amount of product pressure decreases compared to the lower cut.

IV. CONCLUSION

In this work a set of nonlinear equations for cut controlling is developed. The equations can simulate pressure losses, flow rates in a cascade and valve setting. In this method, functions of single machine used to calculate the behavior of the cascade and it is found that for controlling stages cut, putting one valve in the product section of each stage is enough. The cut extracted from the optimum binary mixture or multi component cascade can be as an input of the equations. It is shown that by adjusting the product pressure valves of the cascade, the cuts can reach to desirable value.

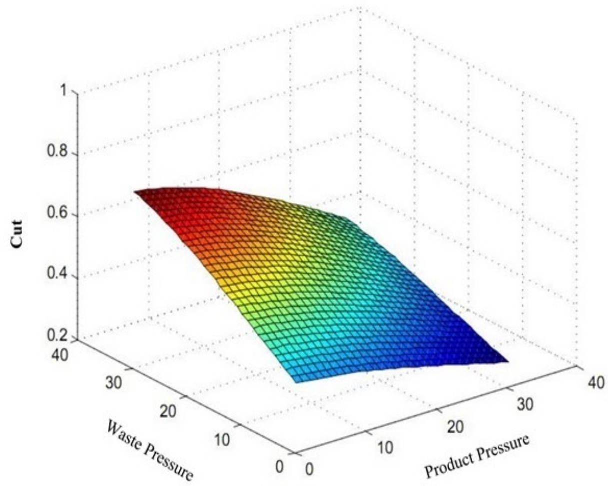

Fig. 3. The cut dependence on the pressure

Table 1

The Parameters and Valve Cross Sections (A) of the Optimum Cascade

cut	Number of machines	Feed pressure(torr)	Product pressure(torr)	A(cm^2)
0.420981	1456	2.0927	5.8755	0.3014
0.428554	2449	2.1248	2.7909	1.1245
0.420302	1374	2.2361	3.3236	0.5235
0.415237	751	2.2578	3.8660	0.2442
0.401767	330	2.2051	4.3945	

Table 2

The Input Parameters of the Cascade

Teta for all stage	0.45 and 0.3
Feed point	7
Cascade feed	8000
Feed of Single	100
Pressure function	$0.02*f_i + 1.3$

Table 3

The Parameters and Valve Cross Sections (A) of the Cascade with 10 Stages and Cut=0.3

cut	Number of machines	Feed pressure(torr)	Product pressure(torr)	A(cm2)
0.3	111	3.2899	8.2427	0.1680
0.3	158	3.2971	8.3260	0.2389
0.3	179	3.2895	8.3326	0.2692
0.3	187	3.2973	8.3256	0.2826
0.3	191	3.2945	8.3328	0.2882
0.3	193	3.2904	8.3302	0.2904
0.3	193	3.2975	8.3265	0.2917
0.3	79	3.2940	8.3330	0.1191
0.3	30	3.2940	8.3298	0.0453
0.3	9	3.2940	8.3298	

Table 4

The Parameters and Valve Cross Sections (A) of the 10-stage Cascade Cut=0.45

cut	Number of machines	Feed pressure	Product pressure	A
0.45	111	3.2823	4.0583	0.1680
0.45	158	3.2998	4.2250	0.2389
0.45	179	3.2943	4.2645	0.2692
0.45	187	3.2983	4.2510	0.2826
0.45	191	3.2948	4.2610	0.2882
0.45	193	3.2957	4.2534	0.2904
0.45	193	3.2963	4.2548	0.2917
0.45	79	3.2999	4.2541	0.1191
0.45	30	3.2880	4.2682	0.0453
0.45	9	3.2865	4.2445	

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