Journal of Nuclear Research and Applications

Research Paper

Vol. 3, No. 1, P 27-33, Winter (2023)

Cited in: https://jonra.nstri.ir

Received: 4 April 2022, Accepted: 13 June 2022

# Shape Evolution and Coexistence in Neutron-Rich Yb, Hf, W, Os, and Pt Nuclei

M. Moazami*, A. Kardan

*School of Physics, Damghan University, B. O. Box:36715-364, Damghan, Iran*

**ABSTRACT**

We have investigated the shape evolution and shape coexistence in the even-even neutron-rich isotopes of Yb, Hf, W, Os, and Pt and the isotones $N = 110\text{-}122$ in the $A\sim190$ within the microscopic-macroscopic Cranked Nilsson-Strutinsky framework. Here, we have focused on the potential energy surface (PES) calculations to detect shape variations with N and Z. The PESs in the ground state for low-lying energy show the presence of collective and non-collective bands. The results are comparable to the studies of others where the pairing correlation was also included. Deformation changes in the isotopic and isotonic chains with increasing the neutron and proton number and the filling of closed shells. The qualitative consistency of our calculations without pairing correlations, especially on the position of the PES's minima, shows that the pairing force has no considerable effect on the nuclear shape, at least in the mass region $A\sim190$.

***Keywords***: Cranked Nilsson-Strutinsky Method; PES Calculations; Deformation Parameters; Shape Coexistence; Shape Evolution

## 1. Introductions

In nuclear physics, nuclear shapes and their evolution play an essential role in understanding nuclear phenomena and describing their structure. Investigating and studying the existence of nuclear shape evolution and their deformation as a function of proton and neutron numbers from different viewpoints has attracted significant theoretical interest [1-15]. Nevertheless, coexisted minima are realized well in the context of the nuclear shell model as arising from intruder excitations, especially in the immediate closed-shell area. Thus, obvious signals of its existence have been obtained around proton or neutron-closed shells [5]. During the last decades, one active and attractive field research has been the shape coexistence phenomena in the $A \sim 190$ mass region (see Refs [2, 5] and references therein). To investigate shape coexistence,, various approximations are available that survey heavy nuclei (About our favorite mass region). Several works have been carried out in Pt, Os, W, Hf, and Yb isotopes chains [9-25], such as Skyrme-function, the Gogny-interaction, or using a relativistic mean-field

*. *Corresponding Author name*: M. Moazami
*E-mail address*: mahnaz.moazami54@gmail.com

approach or interacting boson model (IBM). The neutron-rich nuclei W, Os, and Pt is determined with different shapes in their ground states like prolate, oblate, triaxial, and spherical [19]. The atomic nuclei' ground state shapes evolution is an active and motive research field [19,15]. In general, in the study of the structure of the nucleus, there are two computational methods, microscopic and macroscopic, neither of which alone has been able to describe the nuclei. We aim to use the macroscopic-microscopic Cranked Nilson-Strutinsky (CNS) model [26,27] to describe the shape evolution shape coexistence in the chains of even-even Pt, Os, W, Yb, Hf isotopes in the neutron number range *N =110 - 122* (the proton number *Z=70-78*). The CNS code is an approximate Hartree-Fock method [28]. In this method, a deformed mean field is considered a modified harmonic oscillator for the nuclei. The total energy obtained is minimized relative to the deformation parameters [28]. And the potential energy surfaces (PES), rotational bands, how these energy minimums are located, and their effect on the structure of nuclei are studied.

## 2. Research Theories

In this study, we use the unpaired Cranked Nilson-Strutinsky Model to calculate and study the structure of nuclei and nuclear potential energy surfaces. This approach is a microscopic-macroscopic method, making it one of the best and strongest nuclear models in studying the structure of the nucleus in the different mass regions. This mean-field Hamiltonian described nucleons in the rotating nuclei that it is Cranked modified oscillator Hamiltonian [31],

$$H^{\omega} = H(\varepsilon_2,\gamma,\varepsilon_4) - \omega j_X \quad (1)$$

$H\left(\varepsilon_2,\gamma,\varepsilon_4\right)$ preformed the modified oscillator Hamiltonian in terms of $\left(\varepsilon_2,\gamma,\varepsilon_4\right)$.

$$H\left(\varepsilon_2,\gamma,\varepsilon_4\right) = h_{HO}(\varepsilon_2,\gamma) - \kappa\hbar\omega_0[2l_t.s + \mu(l_t^2 - \langle l_t^2\rangle_N)] + V_4(\varepsilon_4,\gamma) \quad (2)$$

The index t in the orbital angular momentum operator $l_t$ is defined in stretch coordinates [33]. Parameters $\kappa$ and $\mu$ determine the strength of the $l_t.s$ and $l_t^2$ terms [33]. The total microscopic energy is given by the summing of these single-particle energies over the occupied states. That is given by Eq. (3).

$$E_{sp}(\varepsilon_2,\gamma,\varepsilon_4,I) = \sum_{occ} e_i^{\omega}(\varepsilon_2,\gamma,\varepsilon_4) \quad (3)$$

When the number of neutrons and protons changes or the shape of the nucleus varies, a sum of single-particle energies illustrates the fluctuations in the total energy, and this caused Strutinsky to renormalize summation, $E_{sh}$ to empirically learned liquid drop energy $E_{LD}$. Thus, the total energy is obtained by using the shell correction method which is defined as the sum Eq. (4),

$$E_{tot}(\varepsilon_2,\gamma,\varepsilon_4,I) = E_{LD}(\varepsilon_2,\gamma,\varepsilon_4,I) + E_{sh}(\varepsilon_2,\gamma,\varepsilon_4,I) \quad (4)$$

And the macroscopic term $E_{LD}$ is considered by the Lublin Struburg Drop (LSD) formula [27]. The PES is calculated by minimizing the total energy with notice to the hexadecapole parameters and plotted in the $(\varepsilon_2,\gamma)$ plane.

## 3. Calculation

This work minimizes the energy in a three-dimensional deformation space $\left(\varepsilon,\gamma,\varepsilon_4\right)$ in the standard CNS code. The three-dimensional space of the mesh we used in these calculations is as follows (numbers in brackets indicate the step length in the calculation for a given variable):

$$x = 0.00(0.04)0.64$$
$$y = -0.28(0.04)0.32$$
$$\varepsilon_4 = -0.09(0.03)0.09$$

The standard parameter is considered for the mass region of heavy nuclei in the calculations for the single-particle parameters $\kappa$ and $\mu$ in Refs. [29-31]. Where the *(x,y)* coordinates are related to the coordinates *(ε, γ)* [29]. According to the mesh, there are $17\times16\times7=1904$ points in the deformation space that each point defines the deformation [29,32]. The solution of this Hamiltonian is performed in the ground state and one of the quantum configurations $(\pi,\alpha)$ of parity and signature. We are currently examining the ground state of these isotopes. To configure the $(+,+)$ spin parity, the ground state of the nuclei with even-A is $0^+$ and the spin range $(I = 0.5-76.5)$ is obtained. To study the properties and characteristics of a nucleus, we need to know the eigenfunctions, eigenvalues, and total energy, so per point of the mesh, the CNS Hamiltonians must be calculated.

**4. Results and Discussion**

Figure 1 examines the isotopic chain PESs of the Yb, Hf, W, Os, and Pt even-even nuclei with neutron numbers *(110 ≤ N ≤ 120)* using the CNS method. In these diagrams, we can see an increase in the number of neutrons from left to the right and in the number of protons from top to bottom. In this study, we can see interesting results that are comparable to the studies of others [9-15]. As expected, due to the increase in the number of neutrons and protons and the filling of the closed shell, we must have both shape evolution and deformation changes, which are quite evident in the isotopic and isotonic chains of these nuclei. In all PESs in the ground state shape, coexistence was considered for low-lying energies in all isotopes, except in some Pt isotopes, the simultaneous presence of collective and non-collective bands is quite visible. In these nuclei, by increasing Z, the transition from prolate to oblate can show that we have almost the same course in the first three Yb-W nuclei, but the conditions will differ for Os and Pt. In the study of Yb isotopes, we see that this transition occurs at *N = 118*, i.e from *N = 110* to *N = 118*, the Prolate shape is visible. And appears from ${}^{190}_{70}Yb_{120}$ as an oblate shape, also in these diagrams in the ground state can be seen triaxial minimums at higher energy levels and shallower depths for ${}^{184-188}_{70}Yb_{114-118}$. Also, for ${}^{190}_{70}Yb_{120}$ there is a coexistence between the prolate and oblate shapes (the collective and non-collective), only these Prolate minimums are visible at shallow energies. There is also this transition for the second row of the hafnium isotopic chain. These shapes evolution occurs with the appearance of triaxial minima from ${}^{186}_{72}Hf_{114}$ to ${}^{190}_{72}Hf_{118}$. In ${}^{188}_{72}Hf_{116}$ which is the same as the transition nucleus from prolate to oblate. In some studies [9-13], triaxial minima with the same energy can be seen in all three regions $(-120<\gamma<-60)$, $(-60<\gamma<0)$ and $(0<\gamma<60)$. Of course, it can be seen that these minimums are present in its two neighboring nuclei but at higher energies. But the minimum energies of the yrast and close to the yrast from *N =120* appear as an oblate. Nevertheless, it is expected that as the N increases from left to right for all nuclei, the deformation will decline due to the proton (neutron) shell filling and approaching the magic numbers 82 (126). In ${}^{194}_{72}Hf_{122}$ The interesting thing that happens is that the presence of minima with energy close to yrast is greater in difference quadrupole deformations. This, of course, considering them in the yrast state, which it can be seen correctly in the PES diagrams because, as it is obvious, the nuclei tend to have a spherical shape as they approach the closed shell, and the deformation in them decreases. For row three, the tungsten isotopes (196 -184) show the shape transition at the same N = 118 and the triaxial minimums at higher energy levels, and like the previous two nuclei, the oblate bands are formed at *N = 120*. At *N = 122*, it can be seen that the isosceles minima with the yrast state are non-axially extended and the deformation reduction is also observed. An interesting thing is happening for the isotopic chain Os in the fourth row of these diagrams. There is no shape transition or the triaxial minimum in this nucleus. Only in the neutron range *N = 110 - 122*, the two minimums of collective and non-collective prolate coexisted are also evident. We also see that the deformation decreases with the increasing number of neutrons. These rotational and non-rotational bands extend to $^{190}_{76}Os_{114}$and are restricted to the prolate axes again from $^{192}_{76}Os_{116}$and widen at $^{198}_{76}Os_{122}$ and include almost most of the non-axial region. However, at *N = 120*, the two minimums of rotational and non-rotational oblate appear very shallow. Still, no prolate to oblate transitions are evident for the yrast band and close to the yrast. In the Pt isotopic chain, like the isotopic chains expressed, we see a decrease in the quadrupole deformation parameter with increasing the number of neutrons. But the emergence of energy minima in this nucleus, the transition with the first three nuclei, and its neighboring nucleus, Os, is different. Numerous transitions can be observed in this area. There are only triaxial minima in the first three isotopes of platinum N = 110 - 114. And unlike the other four nuclei, there is no Collective and Non-collective coexistence. At *N = 116* we have a transition from non-axial minimums to axial minimums and up to the isotope $^{196}_{78}Pt_{118}$ , we will have coexistence of collective and non-collective prolate shapes. Now we can see the second transition from prolate to oblate at $^{196}_{78}Pt_{120}$ and again we will have a transition at *N = 122* from oblate to prolate. That is, $^{200}_{78}Pt_{122}$, which in this study is the closest nucleus to the closed shell of protons and neutrons, appears with extensive collective and non-collective prolate minima. In contrast to the Osmium nucleus, which in this mass area had no transition for yrast and near the yrast bands, the platinum nucleus behaved quite differently from its neighbor. It can be said that these two nuclei show these current behaviors as they approach the closed shell of protons and neutrons. Also, even though pairing is considered in studies with other interactions, there are interesting and different reports for these two nuclei, especially for platinum [9-15]. Now if we examine the column of these PESs we see that (except in Pt which are only prolate at *N = 116,118*) at *N = 110-118* all isotones are collective and non-collective prolate shape. And at *N = 118* (except in Os) the transition from Prolate to oblate is observed. At N = 120 from top to bottom, even in Os at energies much shallower than the yrast, the simultaneous 4 minimum oblate and prolate collective and non-collective can be seen except Hf and W, which In platinum, these minimums are in energies very close to each other, that is, they are in energies close to yrast. It should be noted that by increasing the number of protons in all isotones, by increasing Z, and by filling the proton shell, we have a decrease in the quadrupole deformation parameter which is quite evident in the calculation. To better compare these PES diagrams, we can refer to the diagram from Ref. [9] calculated with the Gogny D1S interaction.

## 5. Conclusions

In the present research, we have systematically explored the Potential Energy Surfaces (PESs) in various isotopic chains where nuclear shape transitions occur. In addition, we have explored the shape evolution and shape coexistence in Yb, Hf, W, Os and Pt isotopic chains with neutron number *(N = 110 - 122)* by using the microscopic-macroscopic unpaired Cranked Nilsson-Strutinsky

(CNS). Interesting results are observed in five isotopic chains $70 \le Z \le 78$. For all considered nuclei except Os at *N = 118*, there is a prolate to oblate shape transition. Of course, in the paired calculations, this transition was created at *N = 116* [9]. Also, for all except Pt at *N = 110-114*, there is collective and non-collective coexistence. At *N=118*, in these nuclei (even in Os, albeit at very shallow minima), the coexistence of four minimum oblate and prolate collective and non-collective is reported.

In contrast to the Os isotopes, which had no transition in this mass region, in Pt, a triaxial to prolate shape transition is seen at *N = 114*, then an oblate-prolate shape transition at *N = 118* and an oblate-prolate shape transition at *N = 120*. In all isotonic chains, the quadrupole deformation parameter decreases as the shell fills and approaches the closed shell. It is evident that in the theoretical calculations of others, considering the pairing interaction, this issue has also been reported for $\beta$ [9-15]. However, here in the isotope in which the transition takes place, we will see an increase in the amount of axial quadrupole deformation parameter. Based on the behavior of the closed shell and its alignment as well as other theories, it is observed that the intended nuclei tend to be spherical. The qualitative consistency of our calculations without pairing correlations with paired studies, especially on the position of the PES's minima, shows that the pairing force has no considerable effect on the nuclear shape, at least in the mass region *A~190*.

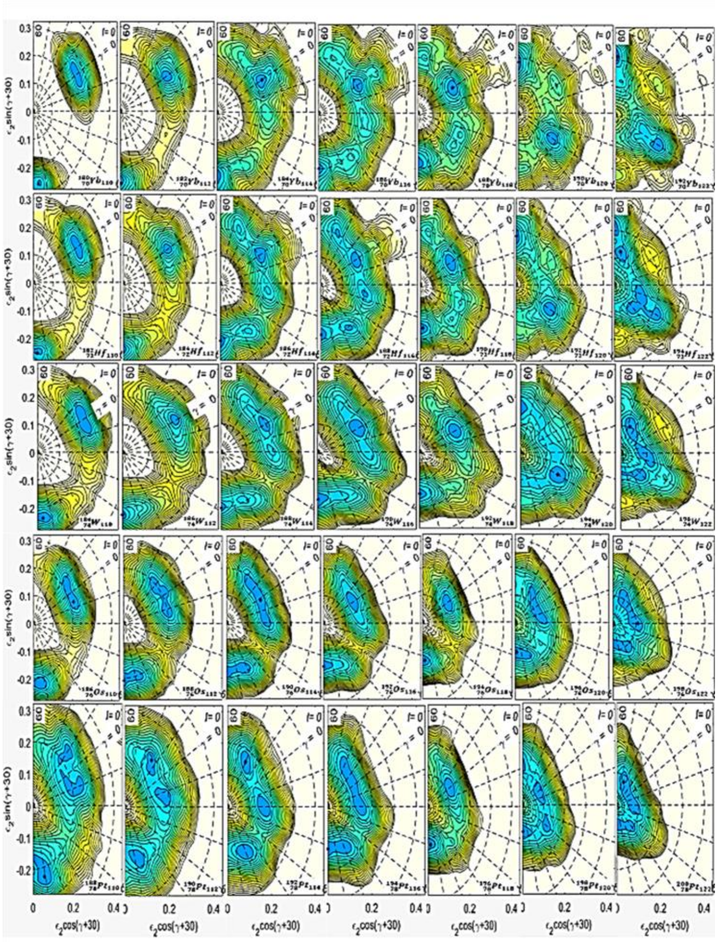

**Fig. 1.** PES diagrams on the *(ε, γ)* plane for even-even isotopic chains Yb, Hf, W, Os, and Pt, respectively, with neutron number *(N = 110-122)*, by using the microscopic-macroscopic unpaired Cranked Nilsson-Strutinsky (CNS) formalism in the ground state $(\hbar\omega = 0)$.

## References


1. R. Wadsworth, et al. "*Evidence for shape coexistence at medium spin in 76Rb*", Physics Letters. B, 701, **3** (2011): 306-312.
2. S. Shohani, and A. Kardan. "*Shape coexistence in even–even Po isotopic chain*", International Journal of Modern Physics E, **28**.10 (2019).
3. P. M. Walker, F. R. Xu, and D. M. Cullen, "*Prolate-oblate shape coexistence at high spin in Hf 175*", Phys Rev C, **71**.6 (2005).
4. Heyde, Kris, and J. L. Wood., Publisher's Note: "*Shape coexistence in atomic nuclei*", Reviews of Modern Physics. **83**, 1467 (2011).
5. García-Ramos, et al. "*Shape evolution and shape coexistence in Pt isotopes: Comparing interacting boson model configuration mixing and Gogny mean-field energy surfaces*", Phys. Rev C, **89**.3 (2014).
6. Wood, J. L., et al. "*Coexistence in even-mass nuclei*", Physics reports, **215**.3-4 (1992): 101-201.
7. Heyde, K., et al. "*Coexistence in odd-mass nuclei*", Physics Reports, 102.5-6 (1983): 291-393.
8. Julin, R., K. Helariutta, and M. Muikku., "*Intruder states in very neutron-deficient Hg, Pb and Po nuclei*", Journal of Physics G: Nuclear and Particle Physics, **27**.7 (2001): R109.
9. Robledo, L. M., R. Rodríguez-Guzmán, and Pedro Sarriguren., "*Role of triaxiality in the ground-state shape of neutron-rich Yb, Hf, W, Os and Pt isotopes*", Journal of Physics G: Nuclear and Particle Physics, **36**.11 (2009): 115104.
10. Sarriguren, Pedro, R. Rodríguez-Guzmán, and L. M. Robledo, "*Shape transitions in neutron-rich Yb, Hf, W, Os, and Pt isotopes within a Skyrme Hartree-Fock+ BCS approach*", Physical Review.C, **77**.6 (2008): 064322.
11. Nomura, K., et al. "*Collective structural evolution in neutron-rich Yb, Hf, W, Os, and Pt isotopes*", Physical Review. C, **84**.5 (2011): 054316.
12. L. M, Robledo, R. R. Rodríguez-Guzmán, and Pedro Sarriguren, "*Evolution of nuclear shapes in medium mass isotopes from a microscopic perspective*", Physical Review C, **78**.3 (2008): 034314.
13. P. Sarriguren, R. R. Rodríguez-Guzmán, and L. M. Robledo, "*Microscopic description of shape evolution in medium-mass nuclei*", Journal of Physics: Conference Series. **205**, 1 (2010).
14. K. Nomura, et al. "*Collective structural evolution in neutron-rich Yb, Hf, W, Os, and Pt isotopes*", Physical Review C, **84**.5 (2011): 054316.
15. K. Nomura, et al. "*Mean-field derivation of the interacting boson model Hamiltonian and exotic nuclei*", Physical review letters, **101**.14 (2008): 142501.
16. García-Ramos, J. Enrique, and K. Heyde, "*The Pt isotopes: Comparing the Interacting Boson Model with configuration mixing and the extended consistent-Qformalism*", Nuclear Physics A, **825**.1-2 (2009): 39-70.
17. Garcia-Ramos, et al. "*Platinum nuclei: Concealed configuration mixing and shape coexistence*", Physical Review C , **84**.1 (2011): 014331.
18. R. Rodríguez-Guzmán, et al. "*Mean field study of structural changes in Pt isotopes with the Gogny interaction*", Physical Review C, **81**.2 (2010): 024310.
19. Zs. Podolyák, et al. "*Weakly deformed oblate structures in 76 198 Os 122*", Physical Review C, **79**.3 (2009): 031305.
20. C. Y. Wu, et al. "*Quadrupole collectivity and shapes of Os, Pt nuclei*", Nuclear Physics A, **607**.2 (1996): 178-234.
21. C. Wheldon, et al. "*Yrast states in 194 Os: The prolate-oblate transition region*", Physical Review C, **63**.1 (2000): 011304.

**22.** A. E. Stuchbery, et al. "*Measured Magnetic Moments and Shape Coexistence in the Neutron-Deficient Nuclei Pt 184, 186, 188*", Physical review letters, **76**.13 (1996): 2246.

**23.** J. P. Rudolf, et al. "*Shape evolution in the neutron-rich osmium isotopes: Prompt γ-ray spectroscopy of Os 196*", Physical Review C , **90**.2 (2014): 021301.

**24.** K. Nomura, et al. "*Interplay between pairing and triaxial shape degrees of freedom in Os and Pt nuclei*", Physical Review C, **104**.2 (2021): 024323.

**25.** P. R John, et al. "*Shape evolution in the neutron-rich Osmium isotopes: Prompt γ-RAY spectroscopy of 196Os*", Physical Review C, **90**.2 (2014): 021301.

**26.** Bengtsson, Tord, and I. Ragnarsson, "*Rotational bands and particle-hole excitations at very high spin*", Nuclear Physics A, **436**.1 (1985): 14-82.

**27.** K. Pomorski, and J. Dudek, "*Nuclear liquid-drop model and surface-curvature effects*" Physical Review C, **67**.4 (2003): 044316.

**28.** L. Rezayi, and A. Kardan. "*Rotational bands in 167Lu at high spins*" School of Physics, Damghan University, (1393).

**29.** A. Kardan, and S. Sayyah, "*Shape evolution with angular momentum in Lu isotopes*", International Journal of Modern Physics E, **25**.06 (2016): 1650044.

**30.** A. Kardan, et al. "*Interpretation of the large-deformation high-spin bands in select A= 158–68 nuclei*", Physical Review C, **86**.1 (2012): 014309.

**31.** S. G. Nilsson and I. Ragnarsson, "*Shapes and Shells in Nuclear Physics*", Cambridge University Press, (1995).

**32.** A. Kardan, "*Nuclear hexacontatetrapole deformation effects on the rotational structures at large deformations in 158Er*", International Journal of Modern Physics E, **26**.07 (2017): 1750044.

**33.** A. Kardan, et al. "*Interpretation of the large-deformation high-spin bands in select A= 158–168 nuclei*", Physical Review C , **86**.1 (2012): 014309.



**How to cite this article**

M. Moazami, A. Kardan, *Shape Evolution and Coexistence in Neutron-Rich Yb, Hf, W, Os, and Pt Nuclei*, Journal of Nuclear Science and Applications, Vol. **3,** No. **1,** P **27-33,** Winter **(2023), Url**: https://jonra.nstri.ir/article_1434.html, **DOI**: 10.24200/jon.2023.1003