Plasma Mismatching with Microwave Transmission Characteristics for Diagnostic Purposes

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ABSTRACT

Many remote diagnostic systems rely on the transmission and reflection of electromagnetic waves. In diagnostics facilities such as microwave reflectometers which operate in the cut-off frequency range, the collisionality of the plasma can affect the transmission and reflection characteristics of electromagnetic waves. In this paper, plasma mismatching and reflection properties of microwave propagation through partially ionized, uniform, and cold plasma were studied. Using the simplified Lorentz model of the plasma and associated dispersion relation of electromagnetic wave propagation through collisional, cold plasma, propagation characteristics of the electromagnetic waves were investigated in different collisional regimes of single-interface and double-interface, bounded plasma. It was found that in the cut-off frequency range, the propagation coefficients of the electromagnetic wave can be significantly affected depending on the collision frequency of the plasma. It was also found that in the double-interface case, the thickness of the plasma can cause multiple reflections in the plasma slab, and such reflections can be suppressed in the high-loss collisional regime.

Keywords: Wave Propagation in Plasma; Collisional Plasma; Microwave Plasma Diagnostics; Partially Ionized Plasma

1. Introductions

Theoretical and experimental investigations of electromagnetic (EM) wave propagation in plasma have been studied extensively because of various applications, including plasma diagnostics, radar-absorbing material, telecommunication systems, radar cross-section reduction, etc. [1, 2, 3, 4, 5, 6, 7]. In many experimental studies of plasma research, it is not possible to obtain basic information and parameters of the plasma by direct diagnostic techniques, and indirect (remote) methods are extensively used [7, 8, 9]. Diagnostic facilities, which are based on the interaction of electromagnetic waves with plasma, play a crucial role in the determination of plasma parameters such as electron density, spatial distribution, and

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fluctuations of the plasma [10, 11, 12]. In addition, diagnostic systems based on the interaction of electromagnetic waves with plasma have the lowest error and the highest resolution of results compared with direct diagnostic tools [13, 14, 15, 16]. Generally speaking, electromagnetic radiation in plasma can be generated by the plasma itself or from an external source that interacts with it. Among these, microwave diagnostic systems are of particular importance in determining the characteristics of different types of laboratory plasmas. Electron density properties of plasma, such as linear averaged density, spatial distribution, and temporal turbulence spectrum, play a significant role in understanding and developing plasma physics in many plasma physics experiments such as ionosphere space plasma, nuclear fusion, and atmospheric pressure plasma [17, 18, 19, 20]. In many microwaves diagnostic facilities which operate in the cut-off frequency range of the plasma, the plasma medium can cause mismatching of the transmitted signal into the cut-off layers [21, 22, 23]. In addition, in partially ionized plasma, inter-particle collisions can seriously affect the transmission and reflection properties of the propagated EM wave of microwave range through the plasma [23, 24, 25, 26]. Therefore, the effect of the collision on microwave propagation is of great importance in the case of medium-density plasma for systems such as microwave reflectometers which operate below the cut-off frequency of the plasma. In this paper, considering the Lorentz physical description of collisional, cold plasma, the near cut-off reflection and transmission characteristics of microwave propagation in partially ionized, uniform, and unmagnetized plasma are studied considering single-interface and double-interface, bounded plasma. Generally speaking, the plasma is considered a conducting, dispersive medium that can cause reflection mismatching. Therefore, in an experimental study of the interaction of microwave with bounded plasma, the plasma medium can behave like a transmission line that reflects back the incident signal of the electromagnetic wave to the power source or antenna. Up to the authors’ knowledge, no numerical simulation study has been conducted on the effect of plasma mismatching on the reflection and transmission of electromagnetic wave. The paper’s organization is as follows: In Sec. 2, the physical model and formulation of the problem are described. Numerical simulation and obtained results are described in Sec. 3, and the conclusion is made in Sec. 4.

2. Physical Formulation of the Problem

To model and formulate the propagation of electromagnetic (EM) waves in partially ionized, collisional plasma, a simplified model of Lorentz plasma can be considered. Generally speaking, in the Lorentz plasma it is supposed that electrons interact with each other due to space-charge forces, and ions and molecules are assumed to be at rest because of their greater weight. The dispersion relation of EM wave propagation in unmagnetized, collisional, cold plasma can be written as [1, 8, 9]

\[
c^2 k^2 = \omega^2 - \frac{\omega \omega_p^2}{(iv + \omega)}
\]

(1)

where \( c \), \( v \), and \( \omega \) are the speed of light, collision frequency, and frequency of the EM wave, respectively. Parameter \( \omega_p \) represents the plasma frequency which is proportional to the plasma density as \( \omega_p \approx 56.42 \sqrt{n_e} \). The complex propagation constant of the EM wave can be defined as

\[
\kappa = \beta - i\alpha
\]

(2)

where parameters \( \alpha \) and \( \beta \) are attenuation and propagation constants of the EM wave. As the dispersion relation of Eq. 1 is in complex form, the
associated complex refractive index of the plasma medium can be introduced as

$$\tilde{\eta} \equiv \frac{c k}{\omega} = \beta - i \frac{\alpha}{k_0}.$$  \hspace{1cm} (3)

the real and imaginary parts of the Eq. 3 are associated with the real refractive and attenuation characteristics of the plasma medium. Using the expression of the dispersion relation in Eq. 1 and the definition of the complex refractive index (Eq. 3), the propagation and attenuation constants of the EM wave can be written as [9, 24, 25]

$$\beta = \frac{k_0}{\sqrt{2}} \left[ \frac{1 - \omega_p^2}{(\omega^2 + \nu^2)} \right]^{1/2} + \left[ \frac{\omega_p^2}{(\omega^2 + \nu^2)} \right]^{1/2}$$

$$\alpha = \frac{k_0}{\sqrt{2}} \left[ \frac{1 - \omega_p^2}{(\omega^2 + \nu^2)} \right]^{1/2} + \left[ \frac{\omega_p^2}{(\omega^2 + \nu^2)} \right]^{1/2} \right]^{1/2}.$$  \hspace{1cm} (4)

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To investigate the reflection and transmission of the EM wave from the cut-off layer of the plasma, two cases of collisional, and homogeneous plasma with sharp boundaries, as shown in Fig. 1, are studied.

In the first case (Fig. 1 (a)), a semi-infinite plasma with single interface of vacuum-plasma is considered. A planar EM wave of the form exp$$\left[i(\omega t - k_0 z)\right]$$ propagates in the z direction in the vacuum and incidents into the plasma medium. Form Maxwell’s equation, the transverse components of the electric and magnetic fields of the EM wave are given as

$$E(z) = \begin{cases} E_0 \exp(-ik_0 z) + E_1 \exp(ik_0 z), & z < 0 \\ E_1 \exp(ik_0 z), & z > 0 \end{cases}.$$  \hspace{1cm} (6)

$$H(z) = \begin{cases} \frac{1}{Z_0} \left[ E_0 \exp(-ik_0 z) - E_1 \exp(ik_0 z) \right], & z < 0 \\ \frac{E_1}{Z_m} \exp(ik_0 z), & z > 0 \end{cases}.$$  \hspace{1cm} (7)

the magnitude of the electric and magnetic fields is related to the complex impedance as

$$\tilde{Z}_m \equiv \frac{E}{H} = \frac{Z_0 \sqrt{\mu_m}}{\tilde{\eta}^2}.$$  \hspace{1cm} (8)

where $$\mu_m$$ is the relative permeability of the plasma medium which is usually assumed to be $$\mu_m = 1$$ and the impedance of the free space is defined as $$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 377 \Omega$$. The electric and magnetic fields of the EM wave should be continuous across the vacuum-plasma interface. The boundary conditions of Electric and magnetic fields are given as

[Diagram of reflection and transmission of electromagnetic wave from a single-interface (a) and double-interface (b) plasma medium.]

Fig. 1.
\[ E_0 + E_r = E_t \]
\[ \frac{1}{Z_0} (E_0 - E_r) = \frac{E_t}{Z_m} \]  
(9)

the reflected and transmitted amplitude of the EM wave are defined as

\[ \tilde{r} \equiv \frac{E_r}{E_0} = \frac{Z_m - Z_0}{Z_m + Z_0} \]  
(10)

\[ \tilde{t} \equiv \frac{E_t}{E_0} = \frac{2Z_m}{Z_m + Z_0} \]  
(11)

It is clear from Eqs. 10 & 11 that when the impedances of two media become equal, no reflection occurs and all of the EM waves transmit through the plasma. To describe the reflected and transmitted power of the EM wave, reflection and transmission coefficients of the single-interface plasma are defined as the square of absolute values of the reflection and transmission amplitudes as

\[ R \equiv |\tilde{r}|^2 = \frac{(k_0 - \beta)^2 + \alpha^2}{(k_0 + \beta)^2 + \alpha^2} \]  
(12)

\[ T = 1 - |\tilde{t}|^2 = \frac{4\beta k_0}{(k_0 + \beta)^2 + \alpha^2} \]  
(13)

As the second case study, the reflection and transmission of EM wave from a double-interface, uniform plasma slab, as is shown in Fig. 1 (b), is considered. Similar to the single-interface case, the electric and magnetic fields of the EM wave are given in the following forms [1, 2, 3, 24]

\[ E_s(z) = \begin{cases} E_0 \exp(-ik_0z) + E_r \exp(ik_0z), & z < 0 \\ E_r \exp(-ik_0z) + E_\ast \exp(ik_\ast z), & 0 \leq z \leq d \\ E_\ast \exp(-ik_\ast z), & z > d \end{cases} \]  
(14)

\[ H_s(z) = \begin{cases} \frac{1}{Z_0} [E_0 \exp(-ik_0z) - E_r \exp(ik_0z)], & z < 0 \\ \frac{1}{Z_m} [E_r \exp(-ik_\ast z) - E_\ast \exp(ik_\ast z)], & 0 \leq z \leq d \\ \frac{E_\ast}{Z_0} \exp(-ik_\ast z), & z > d \end{cases} \]  
(15)

where \( E_r \) and \( E_\ast \) are field amplitudes of the reflected and transmitted wave in the plasma slab, respectively. Considering the continuity conditions of the electric and magnetic fields across the vacuum-plasma (\( z=0 \)) and plasma-vacuum (\( z=d \)) interfaces, the reflection and transmission amplitudes can be derived as

\[ \tilde{r}_s = \frac{\tilde{r} \left[ 1 - \exp(-2i\hat{k}d) \right]}{1 - \tilde{r}^2 \exp(-2i\hat{k}d)} \]  
(16)

\[ \tilde{t}_s = \frac{(1 - \tilde{r}^2) \exp \left[ -i (\hat{k} - k_\ast) d \right]}{1 - \tilde{r}^2 \exp(-2i\hat{k}d)} \]  
(17)

where the lower-script "s" in Eqs. 16 & 17 is related to the plasma slab case. The reflection and transmission coefficients are derived as

\[ R_s = |\tilde{r}_s|^2 = \frac{\tilde{r} \left[ 1 + \exp(-4ad) - 2 \exp(-2ad) \cos(2\beta d) \right]}{1 + \tilde{r}^2 \exp(-4ad) - 2 \tilde{r} \exp(-2ad) \cos(2\beta d - 2\phi)} \]  
(18)

\[ T_s = |\tilde{t}_s|^2 = \frac{\exp(-2ad) \left[ 1 + \tilde{r}^2 - 2 \tilde{r} \exp(2\phi) \right]}{1 + \tilde{r}^2 \exp(-4ad) - 2 \tilde{r} \exp(-2ad) \cos(2\beta d - 2\phi)} \]  
(19)

Where \( \phi \) is the phase angle of the reflection amplitude of the single-interface plasma \( (\tilde{r} = |\tilde{r}| \exp(i\phi)) \) which is given as

\[ \varphi = \tan^{-1} \left( \frac{2\alpha k_0}{k_0^2 - \beta^2 - \alpha^2} \right) \]  
(20)
Eqs. 12, 13, 18 & 19 are the main equations which are used to describe the near cut-off characteristics of the EM wave propagation through the single and double-interface, uniform, collisional, cold plasma.

3. Numerical Simulations and Results
The derived equations in Sec. 2 are simulated numerically to investigate the propagation of EM wave in unmagnetized, partially ionized, single and double-interface plasma slabs. As an example of single plasma medium, the proposed approach can be used in description of wave propagation in diagnostic systems which are based on reflection of the electromagnetic waves. In such systems, the frequency of the incident wave is less than the cutoff frequency of the plasma [21]. Therefore, the electromagnetic wave is reflected at the cutoff layers and the plasma is simplified as a single-interface layer. Single-layer radar-absorbing material (RAM) is another example of single-interface plasma [2, 9]. The diagnostic system of many laboratory plasmas such as density measurement of fluorescent lamps (cold plasma) and microwave interferometry of fusion plasma (hot plasma) which are based on the transmission of electromagnetic waves from the plasma are examples of double-interface plasmas [8].

In the case of weakly ionized plasma, electron-neutral collision describes the most dominant collision type of the plasma. The collision frequency of electron-neutral in cold plasma is expressed as [2, 24]

\[ \nu_{en} = N_0 \sigma_{en} v_{th} \]  \hfill (21)

Where \( v_{th} = \sqrt{\frac{k_B T_e}{m_e}} \), \( \sigma_{en} \) and \( N_0 \) are the thermal velocity of the electron, cross-section of electron-neutral elastic collision and density of gas molecules, respectively. Generally speaking, the density of gas depends on the pressure and temperature of the gas and can be expressed in the following form

\[ N_0 \left[ \text{m}^{-3} \right] \approx 3.3 \times 10^{22} p \left[ \text{Torr} \right] \left( \frac{293}{T_g \left[ \text{K} \right]} \right) \]

Fig. 2 describes electron-neutral collision frequency in terms of the electron temperature for the cold plasma range ( \( 0 < T_e < 10 \text{ eV} \)), considering the experimental data of the elastic cross-sections of the H\(_2\), He & Ar [27, 28].

![Fig. 2. Variations of electron-neutral collision frequency in terms of electron temperature of cold plasma range for three common gases He, H\(_2\) and Ar.](image)

It is found from the figure that maximum collision frequency of He and H\(_2\) gases are respectively \( \nu_{en} \approx 1.3 \text{GHz} \) and \( \nu_{en} \approx 4.1 \text{GHz} \) which means that He and H\(_2\) cold plasmas are in the low-loss and intermediate-loss of collisional regimes. For Ar gas, collision frequency increases linearly with the electron temperature and therefore, for electron temperature \( T_e \approx 10 \text{ eV} \), Ar plasma should be considered in the high-loss collisional regime. To describe attenuation length of the EM wave in the plasma due to collisional effect, the skin-depth parameter is defined as [8, 9]

\[ \delta \equiv \frac{1}{\alpha} \]  \hfill (23)

Variation of the skin-depth parameter in terms of the EM wave frequency is shown in Fig. 3 for
plasma density of \( n_e \approx 10^{12} \text{ cm}^{-3} \) and considering different collisional regimes.

As it is clear from Fig. 3, depending on the frequency range of the EM wave, three interaction regions can be distinguished. The first frequency range which is concerned with the \( \omega \& \nu_m \ll \omega_p \), is known as the low-frequency or conducting region. The conductivity of the plasma in the low-frequency range is large and plasma behaves like a conductor. The second region, which is known as cut-off domain, is related to the \( \nu_m < \omega < \omega_p \) and is associated with the intermediate frequency range. Since the frequency of the EM wave is near the cut-off frequency of the plasma, the intermediate range is so important for preliminary design of diagnostic systems such as microwave reflectometers which operate in the cut-off frequency range. For the low-loss plasma case \((\nu_m = 0.5 \text{ GHz})\), the attenuation length parameter in the cut-off region is almost constant. It is also understood from Fig. 3 that the skin depth of different collision frequencies converges near the cut-off frequency and hence, the effect of collision frequency on the attenuation length of the EM wave is not considerable around the cut-off layer of the plasma. The third region of the frequency range is called the high-frequency or dielectric region. In this region, the frequency of the EM wave is higher than the cut-off frequency of the plasma and plasma is almost transparent to the EM wave and behaves like a dielectric medium. Transmission and reflection coefficients of the single-interface plasma, in terms of the EM wave frequency for different collision frequencies, are shown in Fig. 4.

The density of the plasma is chosen as \( n_e \sim 10^{12} \text{ cm}^{-3} \) and the collision frequencies are assumed to be \( \nu_m = 1 \& 10 \text{ GHz} \) which are associated with the low-loss and high-loss collisional regimes of the plasma, respectively. It is clear from the figure that in the conducting range of the frequency, higher collision frequency has significant effects on the transmission and reflection coefficients. In other words, in the low-frequency range, due to higher collisional loss, considerable part of the EM wave is transmitted...
and therefore, less part of the EM wave can be reflected. For experimental and technical purposes, especially in the microwave diagnostic systems, the plasma medium can be considered as a transmission line and it will be helpful to define the voltage standing-wave ratio (VSWR) parameter as 

$$VSWR \equiv \frac{1 + |\rho|}{1 - |\rho|}$$  \hspace{1cm} (24)

Generally speaking, the VSWR parameter is the ratio between transmitted and reflected waves and measures the efficiency of the transmitting EM signal through the transmission line. The plasma medium can be considered as a mismatched transmission line where part of the incident EM signal is reflected back to the power source or antenna. Therefore, technically speaking, higher values of the VSWR parameter indicate poor transmission and conversely, intense reflection of the EM wave from the plasma. It is found from the figure that in the cut-off range, the VSWR parameter of the low-loss plasma is non-linearly descending but is much larger than the VSWR parameter of the high-loss plasma. This means that the collision frequency can affect the efficiency of the reflection power of the EM wave and therefore, probing measurements of the plasma parameters which are based on the reflection of the EM wave, would be more efficient in the low-collisional regime. It is also understood from the figure that in the high frequency range \( f > f_c \), the VSWR parameter of the low-loss and high-loss plasma are converged. Therefore, in microwave diagnostic systems such as microwave interferometer which rely on transmission of the electromagnetic wave through plasma, as it is expected from Fig. 4, collision frequency does not have significant effect on the mismatching of the plasma medium.

To investigate reflection properties of bounded plasma, propagation of EM wave in double-interface plasma (plasma slab) is simulated numerically as the second case of study. In Fig. 6, transmission and reflection coefficients of the EM wave are described for two cases of the collision frequencies \( \nu_{em} = 1 \text{ GHz} \) and high-loss \( \nu_{em} = 10 \text{ GHz} \) collision frequencies.

As it is obvious from the figure, the transmission coefficient of the dielectric frequency region, decreases significantly for the higher collision frequency case which means that the collisionality of the plasma can have considerable effect on transmission of the EM wave in the plasma slab. It is also deduced from the figure that increasing the thickness of the plasma slab, decreases the transmission coefficient of the EM wave. In addition, effect of the plasma thickness on the
transmission of the EM wave is stronger for the higher loss plasma. To better determine the reflection characteristics of the EM wave from the plasma slab, the decibel scale of the reflection coefficient is defined as

$$ R_{dB} = 10 \log R $$

(25)

In Fig. 6(c) the decibel scale behavior of the reflection power in terms of the EM wave frequency is shown. As it is clear from the figure, several valleys of the reflection power are observed in the cut-off and dielectric regions which are due to finite thickness resonance effect causing from multiple internal reflections of the vacuum-plasma and plasma-vacuum interfaces at \( z=0 \) and \( z=d \).

The \( m \)-th order of the multiple resonance satisfies the equation

$$ \beta = \frac{m \pi}{d} $$

(26)

It is deduced from Eq. 26 and Fig. 6(c) that as the plasma thickness increases, the number of resonance valleys also increases and the amplitude of the valleys becomes deeper for higher frequency ranges. It is also understood from the Figs. 6(b) & 6(c) that for the higher collision frequency, the number of the resonance valleys decreases and amplitude of the valleys are suppressed which means that highly collisional plasma slab, is less affected by the multiple internal reflections of the EM wave.

Similar to the single-interface plasma, the VSWR parameter of the double-interface plasma is defined as

$$ \text{VSWR}_d \equiv \frac{1 + \left| \frac{n_f}{n_s} \right|}{1 - \left| \frac{n_f}{n_s} \right|} $$

(27)

where the lower- script ”s” stands for the plasma slab case. Fig. 7 describes color plots of the VSWR parameter of plasma slab with \( d=10 \) cm thickness versus frequency of the EM wave and normalized density of the plasma (\( \omega_p / \omega = n_e / n_i \)).

As validation of the obtained results, it is found that the results of the Fig. 7 are qualitatively in agreement with the simulations of the VSWR parameter in Ref. [22] which is based on the simulation of plasma sheath using CST microwave studio software. As it is understood from the Fig.
7(a), in the conducting region of the frequency and low-loss collision regime \( (\nu_{\text{cm}} < 1.5 \text{GHz}) \), higher values of the VSWR parameter are found \( (\text{VSWR} \approx 100) \). Therefore, in comparison with the high-loss plasma, larger power of the incident EM wave 7 can be reflected from the cut-off layer of the low-loss, double-interface plasma. As it can be found from the Fig. 7(a), in the low-loss plasma, for the frequency range \( f < 0.5 f_p \), intense reflection of the EM wave can be achieved.

![Fig. 7. Colour plot of voltage standing wave ratio (VSWR) parameter versus collision frequency, frequency of EM wave (a) and normalized electron density \( \left( n_e/n_i \right) \) (b) for double-interface plasma of thickness \( d = 10 \text{cm} \).](image)

The validity of the proposed model of the present study for non-uniform plasma slab can be expressed in several points of view. Many low temperatures and unmagnetized lightning plasmas such as fluorescent and some high intensity arc lamps can be simplified as uniform laboratory plasmas [25]. However, for non-uniform plasma slabs the whole of plasma can be divided into several uniform sub-slabs therefore, propagation of electromagnetic wave in non-uniform plasma slab, can be modeled as interaction of the wave with the multi-layer uniform plasma and hence, connectivity of the electric and magnetic fields can be done using scattering matrix methods (SMM) [1, 29].

4. Conclusion

Reflection and transmission properties of electromagnetic (EM) wave propagation through unmagnetized, partially ionized, and uniform plasma were investigated. Propagation characteristics of the EM wave through two bounded plasma cases, including single and double-interface were simulated numerically, considering Lorentz description of collisional, and cold plasma. It was found that collisionality of the plasma can have significant effects on the reflection and transmission of the EM wave. In addition, intense power of the EM wave can be reflected from the low-loss collisional plasma. It was also found that in the double-interface plasma, the plasma thickness does not have significant effect on the reflection and transmission of the EM wave but can cause multiple reflections from the plasma-vacuum interfaces. It was deduced from simulations that high-loss, collisional plasma can suppress the multiple reflections of the EM wave and intensify transmission power of the EM wave in double-interface plasma. Such analysis of EM wave propagation can be considered in early-stage design of microwave diagnostics systems which are based on the reflection and transmission of the EM wave.
References


