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Study of Ideal and Optimum Cascades Using Co-evolutionary Particle Swarm Optimization Algorithm

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ABSTRACT

Ideal cascades for binary mixtures of isotopes are specified by no-mixing at confluent points and minimum total flows. Studies show that there are another types of cascades called the optimum cascade. These cascades have total flows lower than ideal cascades while separation factors are greater than unity and mixings are allowed. In this paper, using a Co-evolutionary Particle Swarm Optimization (CPSO) algorithm, the ideal and optimum cascades are compared in different operating regimes. The CPSO is a metaheuristic algorithm that uses the concept of co-evolution to deal with constrained engineering optimization problems. With the use of the CPSO algorithm, the weighting coefficients of the objective function are adjusted in a self-tuning manner. In this study, it is used to find the parameters of the optimum cascade. Ideal cascades are first classified into four types based on the various relationships between the number of stages of enriching and stripping sections. Three test cases are considered to compare ideal and optimum cascades. The first test case includes two examples of ideal cascades of symmetrical separation stages. In the first example, the total flow for the ideal type 3 cascade and its corresponding optimum cascade is obtained as $\sum L/P = 176.7128$, and in the second example for the ideal type 1 cascade and its corresponding optimum cascade, it is obtained as $\sum L/P = 202.7828$. The results show that for the ideal cascades of symmetrical separation stages, the ideal cascade coincides with the optimum cascade. In test case 2, the total flow for the ideal type 1 cascade of non-symmetrical separation stages and its corresponding optimum cascade (CPSO) is obtained as $\sum L/P = 477.6170$ and $\sum L/P = 228.6997$, respectively. In test case 3, for the ideal type 2 cascade of non-symmetrical separation stages and its corresponding optimum cascade, the total flows are obtained as $\sum L/P = 299.99$ and $\sum L/P = 191.6584$, respectively. The results show that for ideal cascades of non-symmetrical separation stages, the non-mixing condition does not coincide with the condition of the minimum total flow.

***Keywords*:** Ideal Cascade; Optimum Cascade; Constrained Optimization; CPSO.

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1. Introductions

Since the separative power of a single centrifuge is relatively small, multi-stage separation installations, made up of a huge number of separation elements are used in order to obtain an adequate throughput and enrichment (1). Cohen (2) believed that in an "ideal cascade" mixing of materials with different molar fractions should be avoided (no-mixing condition). Initially, the ideal cascade theory was developed for cascades with small separation factors that was corresponded to the gaseous diffusion method. It was found that in ideal cascades, the total inter-stage flow is minimum [1, 2]. More researches in this field led to the development of a new concept of cascades called the "optimum cascade", which is especially applicable to cascades with large separation factors [3]. Studies showed that these cascades allow mixing but still have a less value of the total flow in comparison to the corresponding ideal cascades [4]. Despite the thermodynamic losses of mixing concentrations at confluent points (merged flows), this seemed a bit strange at first glance. The conclusion that the concepts of the ideal and optimum cascades may not coincide in the general case was first made by Palkin [5]. Extending previous works, Palkin and Frolov [6] studied the suboptimal properties of the ideal cascade with symmetric stages and high separation factors. This immediately raised the question that what is the general correlation between a classical ideal cascade and the possible class of such an optimal cascade. It demonstrated a need for an optimization method to obtain parameters of the optimum cascade.

The problem of cascade optimization is a crucial issue that is still raised today. Many studies have been conducted on the cascade optimization. For example, Norouzi, Minuchehr [7] used a real coded genetic algorithm to optimize the parameters of a countercurrent cascade and minimize the number of centrifuges. The reason for using this technique was its versatility and ability to solve highly non-linear, mixed integer optimization problems. Borisevich, Borshchevskiy [8] compared ideal and optimum cascades with variable overall separation factors considering three different types of gas centrifuges. In some studies, optimization of the cascades for separation of the multicomponent mixtures of isotopes has been investigated [9, 12]. In this regard, Ezazi, Imani [13] applied various nature-inspired paradigms in the overall optimization of square and squared-off cascades to separate a middle isotope of tellurium. Ezazi, Mallah [14] investigated the net cascade using ant colony optimization algorithm in another study. Khooshechin, Mansourzadeh [15] optimized a flexible square cascade for high separation of stable isotopes using an enhanced PSO algorithm.

The main purpose of this paper is to study the ideal and optimum cascades using Co-evolutionary Particle Swarm Optimization (CPSO) algorithm. So far, penalty function methods have been used in various works in the literature, but setting suitable penalty factors has often not been an easy task. This paper proposes the CPSO approach to handle this problem. In CPSO two kinds of swarms are applied in two spaces for evolutionary of both solutions and penalty factors. The choice of this technique for cascade optimization would be based on the following criteria: (i) avoiding from solving the multidimensional derivative equations; (ii) avoiding the errors due to transition from discrete to continuous values in the derivation process; (iii) not using any special approximation or assumption in the optimization process; (iv) the ability to obtain the optimal parameters in the large-scale problems; and (v) the ability to handle optimization problems with complex constraints. To the best of our knowledge, applying a co-evolutionary technique in the cascade optimization has not been considered so far, and this paper is the first one. In this paper using this optimization method, the performance of ideal and optimum cascades is compared in several test cases.

2. The Ideal Cascade Theory and Classification

Consider an ideal countercurrent cascade for separating a binary mixture of uranium isotopes that includes (*N-f+1*) separating stages in the enriching section and (*f-1*) stages in the stripping section. N is the total number of stages in the cascade and f is the feed entry point. In classical cascade isotope separation theory, the necessary and sufficient conditions for cascade ideality are given in the common case as follows [4].

$$\alpha_s = \beta_{s+1},\ s = 1,2,\ldots,N-1 \tag{1}$$

$$\begin{cases}\alpha_1 = \beta_2 = \alpha_3 = \beta_4 = \cdots \\ \beta_1 = \alpha_2 = \beta_3 = \alpha_4 = \cdots\end{cases} \tag{2}$$

where α_s and β_{s+1} are heads and tails separation factors for *s*-th and *(s+1)*-th stages, respectively:

$$\alpha_s = \frac{R'_s}{R_s} = \frac{C'_s/1-C'_s}{C_s/1-C_s} \tag{3}$$

$$\beta_s = \frac{R_s}{R''_s} = \frac{C_s/1-C_s}{C''_s/1-C''_s} \tag{4}$$

The overall separation factor of stage s is defined as:

$$q_s = \frac{R'_s}{R_s} = \frac{C'_s/1-C'_s}{C''_s/1-C''_s} = \alpha_s.\beta_s \tag{5}$$

where $R = \frac{C}{1-C}$ is the abundance ratio and C_s, C'_s, and C''_s are the absolute concentrations of desired component in the feed, enriched and depleted flows, respectively. The no-mixing condition is defined as Eq. (6), which indicates the equality of concentrations at the confluent points [1].

$$C'_{s-1} = C_s = C''_{s+1} \tag{6}$$

For a special case, an ideal cascade can be built with symmetrical separation stages ($\alpha_s = \beta_s$) [16, 17]. Sulaberidze, Borisevich [4] classified the ideal cascades into four types based on the various relationships between the number of stages of enriching and stripping sections. These relationships are given in Eq. (7).

$$\begin{aligned}
&type\ 1: \begin{cases} f-1\ is\ even: & R_P = R_F q^{\frac{N-f+1}{2}}, \\ N-f+1\ is\ even: & R_W = R_F q^{-\frac{f-1}{2}}\beta_1^{-1}, \end{cases}\\
&type\ 2: \begin{cases} f-1\ is\ even: & R_P = R_F q^{\frac{N-f+2}{2}}\beta_1^{-1}, \\ N-f+1\ is\ odd: & R_W = R_F q^{-\frac{f-1}{2}}\beta_1^{-1}, \end{cases}\\
&type\ 3: \begin{cases} f-1\ is\ odd: & R_P = R_F q^{\frac{N-f+1}{2}}, \\ N-f+1\ is\ even: & R_W = R_F q^{-\frac{f}{2}}, \end{cases}\\
&type\ 4: \begin{cases} f-1\ is\ odd: & R_P = R_F q^{\frac{N-f}{2}}\beta_1, \\ N-f+1\ is\ odd & R_W = R_F q^{-\frac{f}{2}}, \end{cases}
\end{aligned} \tag{7}$$

where R_F, R_P, R_W are the relative concentrations of the desired component in the feed, product, and waste flows, respectively. Taking a material balance in every cross section of the cascade, the following system of equations are obtained [4].

$$\theta_s L_s - (1-\theta_{s+1})L_{s+1} = \begin{cases} P, & for\ enricher \\ -W, & for\ stripper \end{cases} \tag{8}$$

$$\begin{cases} \delta'_s = C_{s+1} - C_s = \frac{\alpha_s - 1}{1+(\alpha_s-1)C_s} C_s(1-C_s) \\ \theta_s = \frac{\beta_s - 1}{q-1}[1+(\alpha_s-1)C_s] \\ L_s = \begin{cases} \frac{P(C_P - C_s)}{\theta_s \delta'_s}: & for\ enricher \\ \frac{W(C_s - C_W)}{\theta_s \delta'_s}: & for\ stripper \end{cases} \end{cases} \tag{9}$$

Where L_s and θ_s are the feed and the cut of stage s, respectively and the boundary conditions are Eq. (10).

$$\begin{cases} \theta_N L_N = P \\ (1-\theta_1)L_1 = W \end{cases} \tag{10}$$

This system of equations allows the calculation of non-mixing cascade where the value of the product flow rate of the cascade is given in advance.

3. Co-evolutionary PSO (CPSO)

3.1. The Basic PSO

In the early 1990, various studies were conducted on the social behavior of groups of animals (flocks, herds, etc.). These studies have shown that some animals belonging to a particular group, such as birds, fish, and others, are able to share information in their own groups, and this ability provides significant benefits for survival. Inspired by these studies, Kennedy and Eberhart (18) introduced the Particle Swarm Optimization (PSO) algorithm from the concept of swarm intelligence.

For mathematical modeling of PSO, consider a swarm with *P* particles and *n* dimensions. There is a position vector X_i^t and a velocity vector V_i^t for iteration *t* of particles.

$$X_i^t = \left[x_{i,1}, x_{i,2}, \dots, x_{i,n}\right]^T \tag{11}$$

$$V_i^t = \left[v_{i,1}, v_{i,2}, \dots, v_{i,n}\right]^T \tag{12}$$

These vectors are updated with the respect of dimension j according to the following equations (18):

$$x_{i,j}^{t+1} = X_{i,j}^t + V_{i,j}^{t+1} \tag{13}$$

and

$$V_{i,j}^{t+1} = wV_{i,j}^t + c_1 r_1^t\left(pbest_{i,j} - X_{i,j}^t\right) + c_2 r_2^t\left(gbest_j - X_{i,j}^t\right), \tag{14}$$

$i = 1,2, \dots, P$ and $j = 1,2, \dots, n.$

In Eq. (14), the parameter w is called the inertia factor; c_1 and c_2 are the personal and global learning coefficients; r_1 and r_2 are two random values uniformly distributed in the range of [0,1]; $pbest_i$ is the best position of particle i ever found and $gbest_j$ is the position of the global best particle found so far at time *t*. The main steps of the standard PSO can be briefly described as following steps.

Step 1: Generation of initial population and its evaluation by fitness function;

Step 2: Determination of the best personal memories and the best global memories;

Step 3: Updating the position and velocity of each particle based on Eqs. (13) and (14);

Step 4: Repetition of steps 2-5 until convergence criteria is satisfied;

Step 5: Return "*gbest*" and its objective value and end.

The convergence criteria or stopping conditions mentioned in *step 4* may be one of the following criteria: (i) achieving an acceptable level of response; (ii) elapsing a certain number of iterations or time; (iii) passing a certain number of iterations or a specific time without observing a significant improvement in the response; and (iv) checking a certain number of function evaluations (NFEs).

3.2. The Concept of Co-Evolution and the Mechanism of CPSO

Majority of engineering problems are constrained. A group of meta-heuristic methods that are suitable for constrained optimization problems are co-evolutionary algorithms. There is a paradigm called co-evolution behind these algorithms, which means two processes are necessary for self evolution. For this purpose a process is assigned to find the best answers and minimize or maximize the main objective function, and another process is assigned to make the answers feasible. So, there are two phenomena that evolve simultaneously and have a direct effect on each other [19, 20]. Generally, a constrained optimization problem (For instance, a minimization problem) could be described as follows [19]:

$$find\ x\ to\ minimize\ f(x) \tag{15}$$

$$subject\ to: \begin{cases} g_i(x) \leq 0, & i = 1,2,\dots,n \\ h_j(x) = 0, & j = 1,2,\dots,p \end{cases} \quad (16)$$

where x is the vector of decision variables, n is the number of inequality constraints, and p is the number of equality constraints. Please note that equality constraints can also be converted to inequality constraints. The easiest way to deal with such issues is to use the penalty function because of its simplicity and ease of implementation, but tuning suitable penalty factors is not an easy task and requires a lot of trial and error and hand tuning tests. Co-evolutionary algorithms can be used to solve this problem by designing an adaptive mechanism. One of the algorithms that implement the concept of co-evolution to adjust the penalty factors in a self-tuning manner is the co-evolutionary particle swarm optimization. This algorithm was introduced by He and Wang [19] in 2007. In CPSO, two kinds of swarms, using their own exploration and exploitation, are applied to evolve the solution of the problem in two spaces. In the first space, one kind of swarms (so-called meta-algorithm) denoted by $swarm_2$, with the size of M_2, is used to evolve the penalty factors, while in the another space, the second kind of multiple swarms (denoted by $swarm_{1\,j}, j = 1,2,\dots M_2$) with the size of M_1 are applied to evolve the solution decision variables and there is a co-evolution between two kinds of swarms at the same time. So, there are a total number of $M_2 + 1$ swarms to be executed. Each $swarm_{1,j}$ with a certain number of generations (G_1) evolves using the B_j particles from $swarm_2$ for solution evaluation and obtaining a new $swarm_{1,j}$ [19]. The graphical illustration for the notion of co-evolution is shown in Fig. 1.

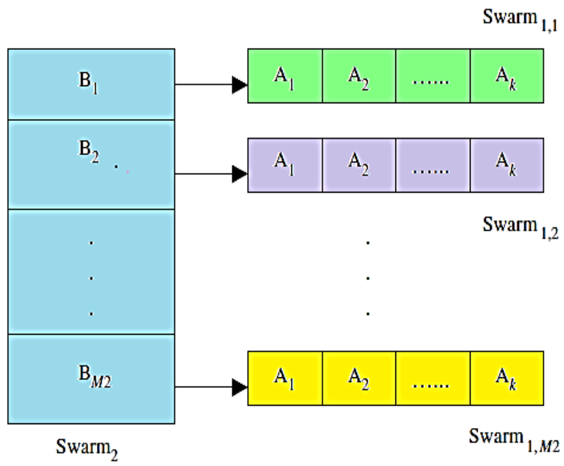


Fig. 1. The principle of co-evolution model in CPSO.

Each particle B_j in $swarm_2$ represents the penalty factors, and each particle A_1 to A_k in $swarm_{1,j}$ represents the decision variable vector of the main problem. Both kinds of swarms apply Eqs. (13) and (14) to obtain the best decision variables. In brief, the two kinds of swarms evolve interactively as long as the cessation condition is satisfied, and finally, not only the decision variables of the main problem are discovered evolutionarily, but also the penalty factors are adjusted in a self-tuning manner.

3.2.1. Evaluation Function of Internal PSO

The evaluation function for the *ith* particle in $swarm_{1\,i}$ is defined as follows [19];

$$F_i(x) = f_i(x) + w_1 \times sum\_viol + w_2 \times num\_viol \quad (17)$$

where $f_i(x)$ is the objective value of the *ith* particle, w_1 and w_2 denote the penalty factors of particle B_j in $swarm_2$. The *sum_viol* and *num_viol* indicate the total sum of violations of constraints and the number of violations, respectively.

3.2.2. Evaluation Function of External PSO

The $swarm_2$ evaluation function is defined according to the following two conditions: **1.** If there is at least one feasible solution in $swarm_{1,j}$, then particle B_j is evaluated using the following formula and is called a valid particle [19]:

$$P(B_j) = \frac{\sum f_{feasible}}{num\_feasible} - num\_feasible \quad (18)$$

where $\sum f_{feasible}$ denotes the sum of objective function values of feasible solutions in $swarm_{1,j}$, and $num\_feasible$ is the number of feasible solutions in $swarm_{1,j}$.

2. If there is no feasible solution in $swarm_{1,j}$, then particle B_j in $swarm_2$ is evaluated as follows and is called an invalid particle [19].

$$P(B_j) = \max(P_{valid}) + \frac{\sum sum\_viol}{\sum num\_viol} + \sum num\_viol \quad (19)$$

where $\max(P_{valid})$ denotes the maximum fitness value of all valid particles in $swarm_2$, $\sum sum\_viol$ denotes the sum of the violations of constraints for all particles in $swarm_{1,j}$, and $\sum num\_viol$ counts the total number of violations of constraints for all particles in $swarm_{1,j}$.

3.3. The Framework of CPSO

Once the mechanism of CPSO algorithm is clear, the framework of the algorithm can be represented by an appropriate flowchart.

4. The procedure of searching for optimal parameters of the cascade

In this section a countercurrent symmetrical cascade for separation of binary mixtures of isotopes is considered. Schematic drawing of this

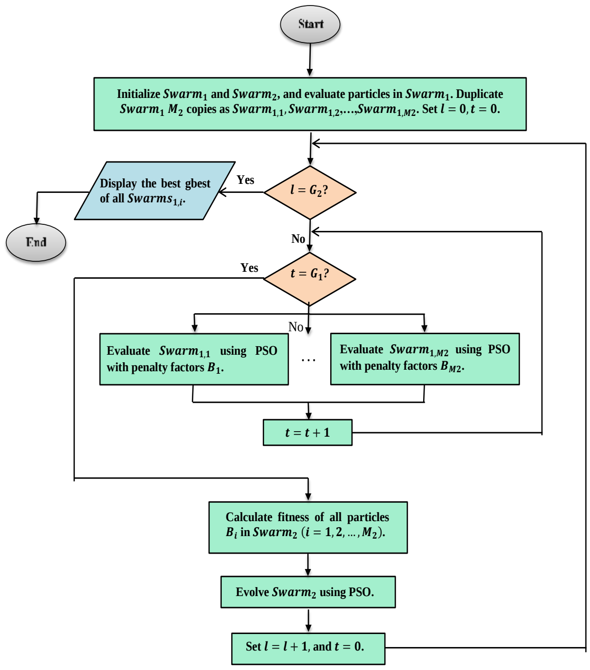


Fig. 2. Flowchart of the CPSO algorithm.

cascade is shown in Fig. 3. The external parameters of the cascade are cascade feed (F), cascade product (P), and cascade waste (W) flow rates, and the corresponding concentrations of desired isotope in these flows, C_F, C_P, and C_W. The cascade consists of a total number of N stages in which the feed flow enters at stage f. The internal parameters of the cascade are the feed, heads and tails flow rates of each stage, L_s, L'_s, and L''_s, and their corresponding concentrations C_s, C'_s, and C''_s. The cut of each stage is defined as $\theta_s = \frac{L'_s}{L_s}$ and q_s denotes the overall separation factor of stage s.

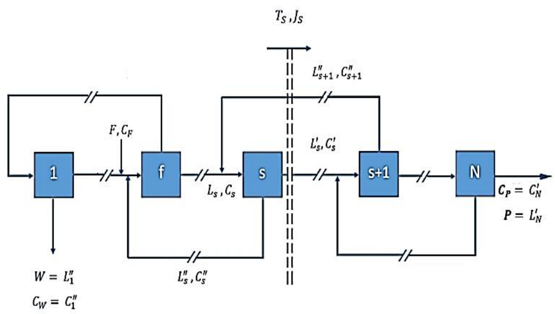


Fig. 3. Schematic drawing of a countercurrent symmetric cascade.

In this cascade, the material and component balance equations for an arbitrary stage s can be written as Eqs. (20)-(23) and (24)-(26) [7, 10]. The Eqs. (20)-(26) should be evaluated accompanied with the boundary conditions (27) and (28).

$$L'_s = \theta_s L_s, \ \ s = \overline{1, N}. \tag{20}$$

$$L''_s = (1 - \theta_s) L_s, \ \ s = \overline{1, N}. \tag{21}$$

$$L_s = L'_{s-1} + L''_{s+1}, \ \ s = \overline{2, N-1}. \tag{22}$$

(23)

$$L_1 = L''_2, \dots, L_f = L'_{f-1} + L''_{f+1} + F, \dots, L'_{N-1} = L_N, \ \ for\ feed\ and\ withdrawal\ points.$$

$$L_s C_s = L'_s C'_s + L''_s C''_s, \ \ s = \overline{1, N}. \tag{24}$$

$$L_s C_s = L'_{s-1} C'_{s-1} + L''_{s+1} C''_{s+1}, \ \ s = \overline{2, N-1}. \tag{25}$$

(26)

$$C_1 = C''_2, \dots, L_f C_f = L'_{f-1} C'_{f-1} + L''_{f+1} C''_{s+1} + F C_F, \dots, C'_{N-1} = C_N, \ \ for\ feed\ and\ withdrawal\ points.$$

$$\begin{cases} L''_1 = (1 - \theta_1) L_1 = W, \\ \quad C''_1 = C_W. \end{cases} \tag{27}$$

$$\begin{cases} L'_N = \theta_N L_N = P, \\ \quad C'_N = C_P. \end{cases} \tag{28}$$

It is clear that the value of the net transit flows of the separating substance as a whole, $T_s = L'_{s-1} - L''_s$, and desired component, $J_s = L'_{s-1} C'_{s-1} - L''_s C''_s$, are equal to Eqs. (29) and (30), respectively (4).

$$\begin{cases} T_s = -W, \\ J_s = -W C_W, \end{cases} \quad for \quad 1 < s \le f \tag{29}$$

$$\begin{cases} T_s = P, \\ J_s = P C_P, \end{cases} \quad for \quad f < s \le N \tag{30}$$

Now one can obtain the concentration distribution along the cascade using the following recurrent formulas [4, 8];

$$\begin{cases} (I): C'_s = \frac{q_s C''_s}{1 + (q_s - 1) C''_s}, \quad s = \overline{1, N}. \\ (II): C''_s = C'_{s-1} - \frac{J_s - T_s C'_{s-1}}{L''_s} = C'_{s-1} - \frac{J_s - T_s C'_{s-1}}{(1 - \theta_s) L_s}, s = \overline{2, N}. \end{cases} \tag{31}$$

Using the recurrent formula (31), concentrations are found in the enriched and depleted streams. Determining the concentration of $C'_N = C_P$ at the last stage of the cascade terminates the calculation procedure.

Now, by defining proper objective function to obtain parameters of the optimal cascade, comparison with the ideal cascade can be carried out. The objective function in terms of the mathematical expressions can be expressed as Eqs. (32) and (33).

$$minimize\ f(L_s) = \sum_s L_s \tag{32}$$

$$subject\ to: \begin{cases} C_P = C_P^* \\ C_W = C_W^* \end{cases} \tag{33}$$

where C_P^* and C_W^* are the target isotope concentrations in the product and waste streams of the cascade. Note that $f(L_s)$ corresponds to $f_i(x)$ in Eq. (17). The problem has two equality constraints and, in this situation, the term "*sum_viol*" in Eq. (17) can be calculated by Eq. (34).

$$sum\_viol = \max\left(0, \left|\frac{C_P}{C_P^*} - 1\right|\right) + \max\left(0, \left|\frac{C_W}{C_W^*} - 1\right|\right) \tag{34}$$

The sum of the violations is calculated as the absolute value of the relative error of the constraints. The main objective is to determine the optimal parameters of $\vec{X} = [\theta_1, \theta_2, \dots, \theta_N, f]$ in such a way that the total flow of the cascade is minimized and the constraints $C_P = C_P^*$ and $C_W = C_W^*$ hold on. The calculation algorithm of the cascade modeling is shown in Fig. 4 as a flowchart.

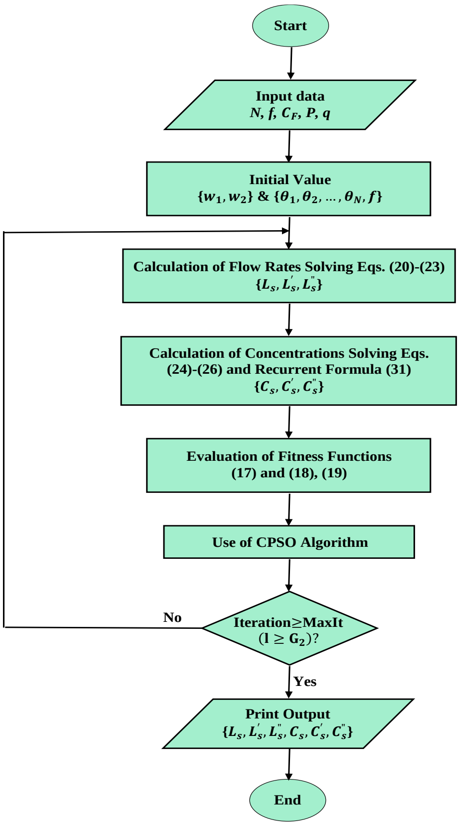


Fig. 4. Calculation algorithm of the cascades modeling.

5. Comparison of the Ideal and Optimum Cascades and Discussion

5.1. General Parameters of CPSO

The general performance of CPSO has been evaluated in [19]. In this paper the CPSO is evalauted on ideal and optimum cascade problems. The general parameters of CPSO for each testing problem are set as follows. The personal and global learning coefficients and inertia factor for both two kinds of swarms are set as: $c_1 = c_2 = 1{,}4962$, and $w = 0{,}7298$; the maximum and minimum velocities for particles in both two kinds of swarms are set as: $V_{i\,max} = 0{,}1 \times (X_{i\,max} - X_{i\,min})$ and $V_{i\,min} = -V_{i\,max}$. The maximum number of generations $(G_1\ \ G_2)$, the population size $(M_1\ \ M_2)$, and the lower bounds and upper bounds of particles in $swarm_{1\,j}$ $(X_{i\,min}\ X_{i\,max})$ and $swarm_2$ $(w_{i\,min}\ w_{i\,max}\ \ i = 1\ 2)$ are specified depending on the problem of each test case.

5.2. Numerical Evaluation of Cascades

To compare the ideal and optimum cascades, a series of numerical examples are given in this section. The calculations are performed according to four different types of ideal cascades classified in section 2. The external parameters of the cascades in all of the cases are selected as follows: The concentration of the desired component in the feed flow is $C_F = 0.711\%$; The cascade output product flow rate is $P = 1\,\mathrm{g}/sec$; The target concentration of the desired component in the product flow is $C_P^* = 4.40\%$; and the overall separation factor at all of the cascade stages is $q_s = 1.592$.

5.2.1. Test Case 1: Ideal Cascades of Symmetrical Separation Stages

The first test case includes two examples of ideal cascades of symmetrical separation stages. The first example is an ideal type 3 cascade with the odd number of stages in the stripping section and the even number of stages in the enriching section (*N*=9, *f*=2). The second example is an ideal type 1 cascade with the even number of stages in the both stripping and enriching sections (*N*=10, *f*=3). The separation stages of the cascades work in a symmetrical regime, where the condition $(\alpha = \beta = \sqrt{q})$ is valid. Computational results are presented in Tables 1 and 2, respectively. The values of the total flow are obtained as $\sum L/P = 176.7128$ and $\sum L/P = 202.7828$, respectively. It is revealed that the minimum total flow in the ideal cascades of symmetrical separation stages coincides with the corresponding optimum cascade.

Table 1. Calculated parameters of the ideal type 3 cascade (*N*=9, *f*=2) of symmetrical separation stages and its corresponding optimum cascade using ideal treatment and CPSO.

Stage	*L, g/sec*	θ_s	Concentrations (%)			Separation factors	
			C_s	C_s'	C_s''	α_s	β_s
1	24.8853	0.4228	0.5643	0.7110	**0.4500**	1.2617	1.2617
2	45.1547	0.4430	0.7110	0.8954	0.5643	1.2617	1.2617
3	34.1250	0.4432	0.8954	1.1272	0.7110	1.2617	1.2617
4	25.3760	0.4434	1.1272	1.4180	0.8954	1.2617	1.2617
5	18.4329	0.4438	1.4180	1.7826	1.1272	1.2617	1.2617
6	12.9185	0.4442	1.7826	2.2387	1.4180	1.2617	1.2617
7	8.5334	0.4447	2.2387	2.8082	1.7826	1.2617	1.2617
8	5.0396	0.4454	2.8082	3.5174	2.2387	1.2617	1.2617
9	2.2474	0.4462	3.5174	**4.4000**	2.8082	1.2617	1.2617
$\sum L/P = 176.7128$							

Table 2. Calculated parameters of the ideal type 1 cascade (*N=10*, *f*=3) of symmetrical separation stages and its corresponding optimum cascade using ideal treatment and CPSO.

Stage	*L, g/sec*	θ_s	Concentrations (%)			Separation factors	
			C_s	C_s'	C_s''	α_s	β_s
1	17.8852	0.4427	0.4478	0.5643	**0.3600**	1.2617	1.2617
2	33.0700	0.4428	0.5643	0.7110	0.4478	1.2617	1.2617
3	45.1547	0.4430	0.7110	0.8954	0.5643	1.2617	1.2617
4	34.1250	0.4432	0.8954	1.1272	0.7110	1.2617	1.2617
5	25.3760	0.4434	1.1272	1.4180	0.8954	1.2617	1.2617
6	18.4329	0.4438	1.4180	1.7826	1.1272	1.2617	1.2617
7	12.9185	0.4442	1.7826	2.2387	1.4180	1.2617	1.2617
8	8.5334	0.4447	2.2387	2.8082	1.7826	1.2617	1.2617
9	5.0396	0.4454	2.8082	3.5174	2.2387	1.2617	1.2617
10	2.2474	0.4462	3.5174	**4.4000**	2.8082	1.2617	1.2617
$\sum L/P = 202.7828$							

5.2.2. Test Case 2: Ideal Cascades of Non-Symmetrical Separation Stages

A quite different picture occurs when the separation stages of the cascade do not work in a symmetrical regime. The computational results show that in this case the minimum total flow in the ideal cascade constructed of non-symmetrical separation stages does not coincide with the corresponding optimum cascade. This is illustrated by the calculated parameters of the ideal type 1 cascade and its corresponding optimum cascade for the external parameters given in section 5.2.

Tables 3 and 4 depict the results for the ideal and optimum cascades, respectively.

Table 3. Calculated parameters of the ideal type 1 cascade (*N=10*, *f*=3) of non-symmetrical separation stages using ideal treatment.

Stage	*L, g/sec*	$\boldsymbol{\theta_s}$	Concentrations (%)			Separation factors	
			C_s	C_s'	C_s''	α_s	β_s
1	54.7792	0.8361	0.4478	0.4768	0.3000	1.0650	1.4949
2	61.5519	0.1100	0.4768	0.7110	0.4478	1.4949	1.0650
3	96.1974	0.8363	0.7110	0.7569	0.4768	1.0650	1.4949
4	89.2871	0.1102	0.7569	1.1272	0.7110	1.4949	1.0650
5	54.0610	0.8365	1.1272	1.1995	0.7569	1.0650	1.4949
6	49.7124	0.1104	1.1995	1.7826	1.1272	1.4949	1.0650
7	27.5214	0.8369	1.7826	1.8962	1.1995	1.0650	1.4949
8	27.7774	0.1108	1.8962	2.8082	1.7826	1.4949	1.0650
9	10.7364	0.8374	2.8082	2.9852	1.8962	1.0650	1.4949
10	8.9927	0.1114	2.9852	4.4000	2.8082	1.4949	1.0650
$\boldsymbol{\sum L/P = 477.6170}$							

Table 4. Calculated parameters of the optimum cascade corresponding to test case 2 using CPSO.

Stage	*L, g/sec*	θ_s	Concentrations (%)		
			C_s	C_s'	C_s''
1	19.6174	0.5257	0.3929	0.4768	**0.3000**
2	35.9749	0.4694	0.5014	0.6241	0.3929
3	48.5853	0.4623	0.6497	0.8110	0.5110
4	39.6479	0.4471	0.8169	1.0266	0.6473
5	29.5655	0.4343	1.0342	1.3068	0.8249
6	21.1906	0.4412	1.3142	1.6544	1.0456
7	14.9316	0.4408	1.6642	2.0938	1.3255
8	10.1127	0.4481	2.1146	2.5492	1.6807
9	6.3025	0.4397	2.6950	3.3851	2.1534
10	2.7713	0.3608	3.3864	**4.4037**	2.8122
$\sum L/P = 228.6997$					

CPSO parameters:

- $G_1 = 100$, $M_1 = 20$, $G_2 = 20$, $M_2 = 12$;
- $w_{1,min} = w_{2,min} = 5000$, $w_{1,max} = w_{2,max} = 6000$;
- $w_1 = 5.5508e3$, $w_2 = 5.4644e3$;
- $sum\_viol = 8.3062e-4$;
- *Optimization time=3.6474* min.

The integral parameter ζ is defined to demonstrate the advantage of the optimum cascade compared to the ideal (non-mixing) cascade. This parameter characterizes the relative difference in the total flow of the optimum and ideal cascade and is calculated using the following formula:

$$\zeta = \frac{\sum_s (L_s)_{id} - \sum_s (L_s)_{opt}}{\sum_s (L_s)_{id}} \times 100\% \tag{35}$$

For this test case, the parameter ζ is equal to *51.7%*, and therefore the total flow in the ideal cascade exceeds with a large amount in comparison to the optimum cascade. So, this leads us to conclusion that in the cascades constructed of non-symmetrical separation stages, the non-mixing condition $\alpha_s = \beta_{s+1}$ does not coincide with the condition of the minimum total flow.

5.2.3. Test Case 3: Efficiency Evaluation of Ideal Cascades of Non-Symmetrical Separation Stages

Another integral parameter to demonstrate the advantage of the optimum cascade compared to the ideal one is the cascade efficiency coefficient, which is calculated as the ratio of the separation powers of the ideal and optimum cascades:

$$\eta = \frac{\sum_s (\delta U_s)_{id}}{\sum_s (\delta U_s)_{opt}} \tag{36}$$

Where the numerator and denominator are the sum of the separation powers of all the stages in the ideal and optimum cascade, respectively. δU_s can be calculated by the following formula:

$$\delta U_s = L_s' V(C_s') + L_s'' V(C_s'') - L_s V(C_s) = L_s\left[\theta_s V(C_s') + (1-\theta_s) V(C_s'') - V(C_s)\right] \tag{37}$$

where $V(C) = (2C-1).\ln[C/(1-C)]$ is the value function and other variables are defined beforehand. Non-ideality of the cascade profile and the associated mixing losses are taken into account by η. The calculated parameters of the ideal type 2 cascade and its corresponding optimum cascade are illustrated in Tables 5 and 6, respectively.

Table 5. Calculated parameters of the ideal type 2 cascade (*N*=9, *f*=3) of non-symmetrical separation stages using ideal treatment.

Stage	*L, g/sec*	θ_s	Concentrations (%)			Separation powers	
			C_s	C_s'	C_s''	δU_s, g/sec	$(\delta U_s/L_s).10^2$
1	1.40	0.012	0.45	0.706	0.44	0.0207	1.48
2	75.51	0.981	0.706	0.71	0.45	1.2935	1.71
3	76.39	0.118	0.71	1.12	0.706	1.1329	1.48
4	43.22	0.982	1.12	1.13	0.71	0.7390	1.71
5	42.82	0.118	1.13	1.78	1.12	0.6364	1.48
6	21.97	0.982	1.78	1.79	1.13	0.3748	1.70
7	21.72	0.118	1.79	2.80	1.78	0.3241	1.49
8	8.56	0.982	2.80	2.82	1.79	0.1454	1.70
9	8.40	0.119	2.82	4.40	2.80	0.1261	1.50
$\sum L/P = 299.99$						$\sum_s(\delta U_s) = 4.7929$	$\sum_s(\delta U_s/L_s) = 0.1425$

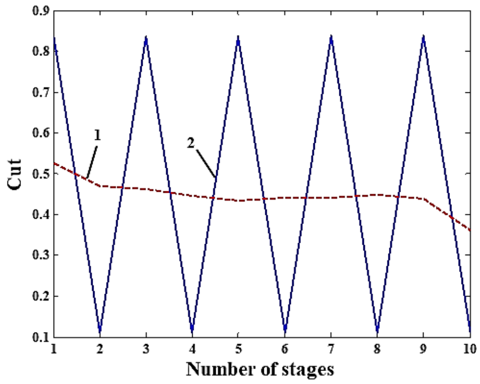
Table 6. Calculated parameters of the optimum cascade corresponding to test case 3 using CPSO.

Stage	*L, g/sec*	θ_s	Concentrations (%)			Separation powers	
			C_s	C_s'	C_s''	δU_s, g/sec	$(\delta U_s/L_s).10^2$
1	20.6497	0.3008	0.5178	0.6987	**0.4400**	0.4955	2.3995
2	34.5759	0.4028	0.6402	0.8218	0.5187	0.9184	2.6560
3	46.8615	0.3947	0.7726	0.9952	0.6275	1.2393	2.6445
4	31.2438	0.4400	1.0065	1.2686	0.8006	0.8402	2.6891
5	22.6274	0.4366	1.2830	1.6187	1.0229	0.6080	2.6872
6	15.8414	0.4395	1.6381	2.0623	1.3054	0.4259	2.6885
7	10.6269	0.4390	2.0929	2.6329	1.6702	0.2857	2.6880
8	6.4486	0.4316	2.6818	3.3811	2.1508	0.1730	2.6831
9	2.7833	0.3593	3.3810	**4.3999**	2.8097	0.0716	2.5717
		$\sum L/P = 191.6584$				$\sum_s(\delta U_s) = 5.0576$	$\sum_s(\delta U_s/L_s) = 0.23708$

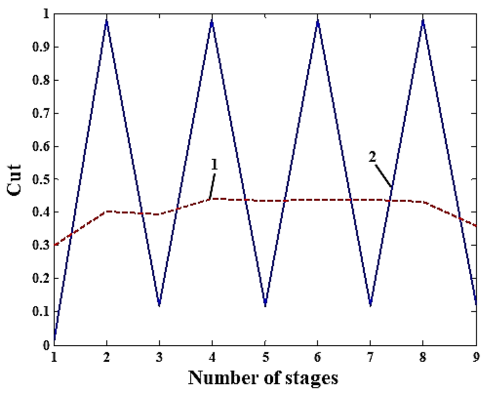
CPSO parameters:

- $G_1 = 100$, $M_1 = 20$, $G_2 = 20$, $M_2 = 12$;
- $w_{1,min} = w_{2,min} = 6000$, $w_{1,max} = w_{2,max} = 7000$;
- $w_1 = 6.5438e3$, $w_2 = 6.4353e3$;
- $sum\_viol = 2.8425e-5$;
- *Optimization time=5.6063* min.

Analyzing the results presented in Tables 5 and 6, one can obtain the cascade efficiency coefficient as $\eta = \sum_s (\delta U_s)_{id} / \sum_s (\delta U_s)_{opt} = 0.9477$ and the parameter $\eta . \sum_s (\delta U_s/L_s)_{opt} / \sum_s (\delta U_s/L_s)_{id} = 1.5766$. Therefore, it is shown that despite the fact that the cascade efficiency coefficient is less than unity, the sum of the specific separation powers in the optimum cascade becomes significantly higher in comparison with that for an ideal cascade. Figures 5-a and 5-b show the cut changes over the cascade stages in test cases 2 and 3, respectively.



a. Test case 2



b. Test case 3

Fig. 5. The cut changes over the cascade stages for the optimum (1) and ideal (2) cascades.

As the figures show, the cut variations in the optimum cascade are smoother than the serrated distribution of the cuts in the corresponding non-mixing cascade constructed of asymmetrical separation stages. It means that all separation stages in the optimum cascade work more effectively, and therefore the total flow in such an optimal cascade becomes lower considerably in contrast to the ideal one. Moreover, a comparison of the cuts in both cascades shows that the cut values in the optimum cascade are more attractive from a technological point of view, due to the slightly varying over the cascade stages.

6. Conclusion

In this paper, a comparative study of ideal and optimum cascades was performed using an effective co-evolutionary particle swarm optimization algorithm. All ideal cascades were divided into four groups that characterize the various relationships between the number of stages of enriching and stripping sections. The CPSO algorithm was used to compare the ideal and optimum cascades. It was shown that the total flow in an ideal cascade, in which the condition $(\alpha = \beta = \sqrt{q})$ is valid for all separation stages, coincides with the total flow in the optimum cascade for arbitrary values of q close to unity. For overall separation factors considerably higher than unity, the distinction between ideal and optimal cascade is essential. For the ideal cascades of asymmetrical separation stages, the non-mixing condition does not coincide with the condition of the minimum total flow. In this case, the condition of the optimum work of a single separation element prevails over the non-mixing condition that leads to divergence in the values for the total flows in ideal and optimum cascades.

References

1. Benedict M, Pigford TH, Levi HW. Nuclear chemical engineering. 2nd ed. New York: McGraw-Hill Education; 1981.

2. Cohen KP. The Theory of isotope separation as applied to the large-scale production of U235. 1st ed. New York: McGraw-Hill; 1951.
3. Palkin VJAe. Optimization of a cascade with arbitrarily specified separation coefficients of the stages. At Energy. 1997;82(4):288-93. doi: 10.1007/BF02418953.
4. Sulaberidze G, Borisevich V, Wood HJSS, Technology. Ideal and optimum cascades. Sep Sci Technol. 2008;43(13):3377-92. doi: 10.1080/01496390802282354.
5. Palkin VJAe. Determination of the optimal parameters of a cascade of gas centrifuges. At Energy. doi: 1998;84(3):190-5.
6. Palkin V, Frolov EJAE. Suboptimal properties of an ideal cascade with symmetric steps. At Energy. 2005;99(3):620-5. doi: 10.1007/s10512-005-0256-9.
7. Norouzi A, Minuchehr A, Zolfaghari A, Hagighattalab AJSS, Technology. Parameters optimization of a counter-current cascade based on using a real coded genetic algorithm. Sep Sci Technol. 2011;46(14):2223-30. doi: 10.1080/01496395.2011.595473.
8. Borisevich V, Borshchevskiy M, Zeng S, Jiang DJCes. On ideal and optimum cascades of gas centrifuges with variable overall separation factors. Chem Eng Sci. 2014;116:465-72. doi: 10.1016/j.ces.2014.05.031.
9. Ezazi F, Mallah MH, Sabet JK, Norouzi AJPiNE. A new method for multicomponent mixture separation cascade optimization using artificial bee colony algorithm. Prog Nucl Energy. 2020;124:103371. doi: 10.1016/j.pnucene.2020.103371.
10. Mansourzadeh F, Khamseh AG, Safdari J, Norouzi AJPiNE. Utilization of harmony search algorithm to optimize a cascade for separating multicomponent mixtures. Prog Nucl Energy. 2019;111:165-73. doi: 10.1016/j.pnucene.2018.11.005.
11. Palkin VJAE. Optimization of a centrifuge cascade for separating a multicomponent mixture of isotopes. At Energy. 2013;115(2):109-15. doi: 10.1007/s10512-013-9757-0.
12. Safdari J, Norouzi A, Tumari RJSS, Technology. Using a real coded PSO algorithm in the design of a multi-component countercurrent cascade. Sep Sci Technol. 2017;52(18):2855-62. doi: 10.1080/01496395.2017.1375524.
13. Ezazi F, Imani M, Safdari J, Mallah MH, Mirmohammadi SLJAoNE. An application of nature-inspired paradigms in the overall optimization of square and squared-off cascades to separate a middle isotope of tellurium. Ann Nucl Energy. 2022;171:109033. doi: 10.1016/j.anucene.2022.109033.
14. Ezazi F, Mallah MH, Sabet JK, Norouzi A, Mahmoudian AJPiNE. Investigation on the net cascade using ant colony optimization algorithm. Prog Nucl Energy. 2020;119:103169. doi: 10.1016/j.pnucene.2019.103169.
15. Khooshechin S, Mansourzadeh F, Imani M, Safdari J, Mallah MHJPiNE. Optimization of flexible square cascade for high separation of stable isotopes using enhanced PSO algorithm. Prog Nucl Energy. 2021;140:103922. doi: 10.1016/j.pnucene.2021.103922.
16. Olander DRJNS, Engineering. Design of ideal cascades of gas centrifuges with variable separation factors. Nucl Sci Eng. 1976;60(4):421-34. doi: 10.13182/NSE76-A26903.
17. Yamamoto I, Kanagawa AJJoNS, Technology. Separative power of ideal cascade with variable separation factors. J Nucl Sci Technol. 1978;15(6):426-32. doi: 10.1080/18811248.1978.9735530.
18. Kennedy J, Eberhart R, editors. Particle swarm optimization. Proceedings of ICNN'95-international conference on neural networks; 1995. Perth, WA: IEEE.
19. He Q, Wang LJEaoai. An effective co-evolutionary particle swarm optimization for constrained engineering design problems. Eng Appl Artif Intell. 2007;20(1):89-99. doi: 10.1016/j.engappai.2006.03.003.
20. Kou X, Liu S, Zhang J, Zheng WJC, Applications Mw. Co-evolutionary particle swarm optimization to solve constrained optimization problems. Comput Math Appl. 2009;57(11-12):1776-84. doi: 10.1016/j.camwa.2008.10.013.

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